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As usual, we assume that the period utility function $u\left(c^{i}\right)$ is strictly increasing in consumption and strictly concave: $u^{\prime}\left(c^{i}\right)>0$ and $u^{\prime \prime}\left(c^{i}\right)<0$.
Let $y^{i}$ denote the individual's output and $r$ the real interest rate for borrowing or lending in the world capital market on date 1 . Then consumption must be chosen subject to the lifetime budget constraint

$$
\begin{equation*}
c_{1}^{i}+\frac{c_{2}^{i}}{1+r}=y_{1}^{i}+\frac{y_{2}^{i}}{1+r} \tag{2}
\end{equation*}
$$

This constraint restricts the present value of consumption spending to equal the present value of output. Output is perishable and thus cannot be stored for later
consumption. ${ }^{2}$
We assume, as we shall until we introduce uncertainty about future income in Chapter 2, that the consumer bases decisions on perfect foresight of the future. This is an extreme assumption, but a natural one to make whenever the complexities introduced by uncertainty are of secondary relevance to the problem being studied. Perfect foresight ensures that a model's predictions are driven by its intrinsic logic rather than by ad hoc and arbitrary assumptions about how people form expectations. Unless the focus is on the economic effects of a particular expectational assumption per se, the deterministic models of this book therefore assume perfect
foresight. ${ }^{3}$ foresight. ${ }^{3}$
To solve the problem of maximizing eq. (1) subject to eq. (2), use the latter to substitute for $c_{2}^{i}$ in the former, so that the individual's optimization problem
reduces to reduces to

$$
\max _{c_{1}^{i}} u\left(c_{1}^{i}\right)+\beta u\left[(1+r)\left(y_{1}^{i}-c_{1}^{i}\right)+y_{2}^{i}\right]
$$

The first-order condition for this problem is

> 1. Until further notice, we also assume that
> $\lim _{c^{i} \rightarrow 0} u^{\prime}\left(c^{i}\right)=\infty$.

The purpose of this assumption is to ensure that individuals always desire at least a little consumption
in every period, so that we don't have to add formal constraits of the in every period, so that we don't have to add formal constraints of the form $c^{i} \geq 0$ to the utility maximization problems considered later.
Whenever we refer to the subjective time
Whenever we refer to the subjective time-preference rate in this book, we will mean the parameter $\delta$ such that $\beta=1 /(1+\delta)$, that is, $\delta=(1-\beta) / \beta$.
2. At a positive rate of interest $r$, nobody would want to store output in any case. In section 1.2 we will in capital to be used in producing future output. 3. Even to be used in producing future output.
3. Even under the perfect foresight assumption we may sometimes loosely refer to an individual's "expectation" or (worse) "expected value" of a variable. You should understand that in a nonstochastic environment, these expectations are held with subjective certainty. Only when there is real uncertainty, as in later chapters, are expected values averages over nondegenerate probability distributions.
$u^{\prime}\left(c_{1}^{i}\right)=(1+r) \beta u^{\prime}\left(c_{2}^{i}\right)$,
which is called an intertemporal Euler equation. ${ }^{4}$ This Euler equation, which will recur in many guises, has a simple interpretation: at a utility maximum, the consumer cannot gain from feasible shifts of consumption between periods. A oneunit reduction in first-period consumption, for example, lowers $U_{1}$ by $u^{\prime}\left(c_{1}^{i}\right)$. The consumption unit thus saved can be converted (by lending it) into $1+r$ units of second-period consumption that raise $U_{1}$ by $(1+r) \beta u^{\prime}\left(c_{2}^{i}\right)$. The Euler equation (3) thus states that at an optimum these two quantities are equal.

An alternative and important interpretation of eq. (3) that translates it into language more closely resembling that of static price theory is suggested by writing it as
$\frac{\beta u^{\prime}\left(c_{2}^{i}\right)}{u^{\prime}\left(c_{1}^{i}\right)}=\frac{1}{1+r}$.
The left-hand side is the consumer's marginal rate of substitution of present (date 1) for future (date 2) consumption, while the right-hand side is the price of future consumption in terms of present consumption.

As usual, individual $i$ 's optimal consumption plan is found by combining the first-order condition (3) [or (4)] with the intertemporal budget constraint (2). An important special case is the one in which $\beta=1 /(1+r)$, so that the subjective discount factor equals the market discount factor. In this case the Euler equation becomes $u^{\prime}\left(c_{1}^{i}\right)=u^{\prime}\left(c_{2}^{i}\right)$, which implies that the consumer desires a flat lifetime consumption path, $c_{1}^{i}=c_{2}^{i}$. Budget constraint (2) then implies that consumption in both periods is $\bar{c}^{i}$, where
$\bar{c}^{i}=\frac{\left[(1+r) y_{1}^{i}+y_{2}^{i}\right]}{2+r}$.

### 1.1.2 Equilibrium of the Small Open Economy

We assume that all individuals in the economy are identical and that population size is 1 . This assumption allows us to drop the individual superscript $i$ and to identify per capita quantity variables with national aggregate quantities, which we denote by uppercase, nonsuperscripted letters. Thus, if $C$ stands for aggregate consumption and $Y$ for aggregate output, the assumption of a homogeneous population of size 1 implies that $c^{i}=C$ and $y^{i}=Y$ for all individuals $i$. Our assumed demographics simplify the notation by making the representative individual's first-order conditions describe aggregate dynamic behavior. The Euler equation (3), to take

[^0]one instance, will also govern the motion of aggregate consumption under our convention.

We must keep in mind, however, that our notational shortcut, while innocuous in this chapter, is not appropriate in every setting. In later chapters we reintroduce individually superscripted lowercase quantity variables whenever consumer heterogeneity and the distinction between per capita and total quantities are important.

Since the only price in the model is the real interest rate $r$, and this is exogenously given to the small economy by the world capital market, national aggregate quantities are equilibrium quantities. That is, the small economy can carry out any intertemporal exchange of consumption it desires at the given world interest rate $r$, subject only to its budget constraint. For example, if the subjective and market discount factors are the same, eq. (5), written with $C$ in place of $c^{i}$ and $Y$ in place of $y^{i}$, describes aggregate equilibrium consumption.

The idea of a representative national consumer, though a common device in modern macroeconomic modeling, may seem implausible. There are, however, three good reasons for taking the representative-consumer case as a starting point. First, several useful insights into the macroeconomy do not depend on a detailed consideration of household differences. An instance is the prediction that money-supply changes are neutral in the long run. Second, there are important cases where one can rigorously justify using the representativeagent model to describe aggregate behavior. ${ }^{5}$ Finally, many models in international macroeconomics are interesting precisely because they assume differences between residents of different countries. Sometimes the simplest way to focus on these cross-country differences is to downplay differences within countries.

We have seen [in eq. (5)] that when $\beta=1 /(1+r)$, the time path of aggregate consumption is flat. This prediction of the model captures the idea that, other things the same, countries will wish to smooth their consumption. When the subjective time-preference rate and the market interest rate differ, the motivation to smooth consumption is modified by an incentive to tilt the consumption path. Suppose, for example, that $\beta>1 /(1+r)$ but $C_{1}=C_{2}$. In this case the world capital market offers the country a rate of return that more than compensates it for the postponement of a little more consumption. According to the Euler equation (3), $u^{\prime}\left(C_{1}\right)$ should exceed $u^{\prime}\left(C_{2}\right)$ in equilibrium; that is, individuals in the economy maximize utility by arranging for consumption to rise between dates 1 and 2 . The effects of a rise in

[^1]$r$ on initial consumption and on saving are rather intricate. We postpone discussing them until later in the chapter.

### 1.1.3 International Borrowing and Lending, the Current Account, and the Gains from Trade

Let's look first at how intertemporal trade allows the economy to allocate its consumption over time.

### 1.1.3.1 Defining the Current Account

Because international borrowing and lending are possible, there is no reason for an open economy's consumption to be closely tied to its current output. Provided all loans are repaid with interest, the economy's intertemporal budget constraint (2) is respected. In the special case $\beta=1 /(1+r)$, consumption is flat at the level $C_{1}=C_{2}=\bar{C}$ in eq. (5), but output need not be. If, for example, $Y_{1}<Y_{2}$, the country borrows $\bar{C}-Y_{1}$ from foreigners on date 1 , repaying $(1+r)\left(\bar{C}-Y_{1}\right)$ on date 2 . Whenever date 2 consumption equals output on that date less the interest and principal on prior borrowing-that is, $C_{2}=Y_{2}-(1+r)\left(C_{1}-Y_{1}\right)$-the economy's intertemporal budget constraint obviously holds true.
A country's current account balance over a period is the change in the value of its net claims on the rest of the world-the change in its net foreign assets. For example, in our initial simple model without capital accumulation, a country's firstperiod current account is simply national saving. (In section 1.2 we will see that in general a country's current account is national saving less domestic investment.) The current account balance is said to be in surplus if positive, so that the economy as a whole is lending, and in deficit if negative, so that the economy is borrowing.

Our definition of a country's current account balance as the increase in its net claims on foreigners may puzzle you if you are used to thinking of the current account as a country's net exports of goods and services (where "service" exports include the services of domestic capital operating abroad, as measured by interest and dividend payments on those assets). Remember, however, that a country with positive net exports must be acquiring foreign assets of equal value because it is selling more to foreigners than it is buying from them; and a country with negative net exports must be borrowing an equal amount to finance its deficit with foreigners. Balance-of-payments statistics record a country's net sales of assets to foreigners under its capital account balance. Because a payment is received from foreigners for any good or service a country exports, every positive item of its net exports is associated with an equal-value negative item in its capital accountnamely, the associated payment from abroad, which is a foreign asset acquired. Thus, as a pure matter of accounting, the net export surplus and the capital account surplus sum identically to zero. Hence, the capital account surplus preceded by a
minus sign-the net increase in foreign asset holdings-equals the current account balance.
Despite this accounting equivalence, there is an important reason for focusing on the foreign asset accumulation view of the current account. It plainly shows that the current account represents trade over time, whereas the net exports view draws attention to factors determining gross exports and imports within a single time period. Those factors are far more than unimportant details, as we shall see in subsequent chapters, but to stress them at the outset would only obscure the basic principles of intertemporal trade.
To clarify the concept of the current account, let $B_{t+1}$ be the value of the economy's net foreign assets at the end of a period $t$. The current account balance over period $t$ is defined as $C A_{t}=B_{t+1}-B_{t}$. In general, the date $t$ current account for a country with no capital accumulation or government spending is
$C A_{t}=B_{t+1}-B_{t}=Y_{t}+r_{t} B_{t}-C_{t}$,
where $r_{t} B_{t}$ is interest earned on foreign assets acquired previously. (This convention makes $r_{t}$ the one-period interest rate that prevailed on date $t-1$.)

### 1.1.3.2 Gross National Product and Gross Domestic Product

Equation (6) shows that a country's current account (or net export surplus) is the difference between its total income and its consumption. The national income of an economy is also called its gross national product (GNP) and is measured as the sum of two components: the value of the final output produced within its borders and net international factor payments. Here, these factor payments consist of interest and dividend earnings on the economy's net foreign assets, which are viewed as domestic capital operating abroad. ${ }^{6}$ (In line with the definition of net exports given earlier, a country's earnings on its foreign assets are considered part of its national product despite the fact that this product is generated abroad.) In terms of our formal model, GNP over any period $t$ is $Y_{t}+r_{t} B_{t}$, as just indicated.
The first component of national product, output produced within a country's geographical borders, is called gross domestic product (GDP). In the present model
6. Strictly speaking, national income equals national product plus net unrequited transfer payments from abroad (including items like reparations payments and workers' remittances to family members in other countries). Workers' remittances, which represent a payment for exported labor services, are not truly unrequited and are completely analogous to asset earnings, which are payments for capital services. We will treat them as such in section 1.5. In practice, however, national income accountants usually don't treat remittances as payments for service exports. The term "gross" in GNP reflects its failure to account for depreciation of capital-a factor absent from our theoretical model. When depreciation occurs, net national product (NNP) measures national incone less depreciation. Empirical economists prefer to work with GNP rather than NNP data, especially in international comparisons, because actual national account estimates of depreciation are accounting measures heavily influenced by domestic tax laws. Reported depreciation figures therefore are quite unreliable and can differ widely from country to country. For the United States, a ballpark estimate of annual depreciation would be around 10 percent of GNP.

Table 1.1
GNP versus GDP for Selected Countries, 1990 (dollars per capita)

| Country | GDP | GNP | Percent Difference |
| :--- | ---: | ---: | :--- |
| Australia | 17,327 | 17,000 | -1.9 |
| Brazil | 2,753 | 2,680 | -2.7 |
| Canada | 21,515 | 20,470 | -4.9 |
| Saudi Arabia | 5,429 | 7,050 | 29.9 |
| Singapore | 11,533 | 11,160 | -3.2 |
| United Arab Emirates | 17,669 | 19,860 | 12.4 |
| United States | 21,569 | 21,790 | 1.0 |
| Source: World Bank World Dermer |  |  |  |

Source: World Bank, World Development Report 1992.

GDP is $Y_{t}$. Typically the difference between national and domestic product is a rather small number, but for some countries, those which have amassed large stocks of foreign wealth or incurred substantial foreign debts, the difference can be significant. Table 1.1 shows several of these cases.

### 1.1.3.3 The Current Account and the Budget Constraint in the Two-Period Model

Our formulation of budget constraint (2) tacitly assumed that $B_{1}=0$, making $C A_{1}=Y_{1}-C_{1}$ on the formal model's date 1 (but not in general). By writing constraint (2) as a strict equality, we have also assumed that the economy ends period 2 holding no uncollected claims on foreigners. (That is, $B_{3}=0$. Obviously foreigners do not wish to expire holding uncollected claims on the home country either!) Thus,

$$
\begin{aligned}
C A_{2} & =Y_{2}+r B_{2}-C_{2}=Y_{2}+r\left(Y_{1}-C_{1}\right)-C_{2} \\
& =-\left(Y_{1}-C_{1}\right)=-B_{2}=-C A_{1},
\end{aligned}
$$

where the third equality in this chain follows from the economy's intertemporal budget constraint, eq. (2). Over any stretch of time, as over a single period, a country's cumulative current account balance is the change in its net foreign assets, but in our two-period model with zero initial and terminal assets, $C A_{1}+C A_{2}=$ $B_{3}-B_{1}=0$.
Figure 1.1 combines the representative individual's indifference curves with the intertemporal budget constraint (2), graphed as
$C_{2}=Y_{2}-(1+r)\left(C_{1}-Y_{1}\right)$.
It provides a diagrammatic derivation of the small economy's equilibrium and the implied trajectory of its current account. (The figure makes no special assumption about the relation between $\beta$ and $1+r$.) The economy's optimal consumption choice is at point $\mathbf{C}$, where the budget constraint is tangent to the highest


Figure 1.1
Consumption over time and the current account
attainable indifference curve. The first-period current account balance (a deficit in Figure 1.1) is simply the horizontal distance between the date 1 output and consumption points. As an exercise, the reader should show how to read from the figure's vertical axis the second-period current-account balance.

Economic policymakers often express concern about national current account deficits or surpluses. Our simple model makes the very important point that an unbalanced current account is not necessarily a bad thing. In Figure 1.1, for example, the country clearly does better running an unbalanced current account in both periods than it would if forced to set $C_{1}=Y_{1}$ and $C_{2}=Y_{2}$ (the autarky point $\mathbf{A}$ ). Intertemporal trade makes possible a less jagged time profile of consumption. The utility gain between points $\mathbf{A}$ and $\mathbf{C}$ illustrates the general and classic insight that countries gain from trade.

## Application: Consumption Smoothing in the Second Millennium B.C.

An early anecdote concerning the consumption-smoothing behavior underlying this chapter's model comes from the story of Joseph in the Book of Genesis. Scholars of the biblical period place the episode somewhere around 1800 b.c.

The Pharaoh of Egypt summoned Joseph, then an imprisoned slave, to interpret two dreams. In the first, seven plump cattle were followed and devoured by seven
lean, starving cattle. In the second, seven full ears of corn were eaten by seven thin ears. After hearing these dreams, Joseph prophesied that Egypt would enjoy seven years of prosperity, followed by seven of famine. He recommended a consumptionsmoothing strategy to provide for the years of famine, under which Pharaoh would appropriate and store a fifth of the grain produced during the years of plenty (Genesis 41:33-36). According to the Bible, Pharaoh embraced this plan, made Joseph his prime minister, and thereby enabled Joseph to save Egypt from starvation.

Why did Joseph recommend storing the grain (a form of domestic investment yielding a rate of return of zero before depreciation) rather than lending it abroad at a positive rate of interest? Cuneiform records of the period place the interest rate on loans of grain in Babylonia in a range of 20 to 33 percent per year and show clear evidence of international credit transactions within Asia Minor (Heichelheim 1958, pp. 134-135). At such high interest rates Egypt could have earned a handsome return on its savings. It seems likely, however, that, under the military and political conditions of the second millennium B.C., Egypt would have found it difficult to compel foreign countries to repay a large loan, particularly during a domestic famine. Thus storing the grain at home was a much safer course. The model in this chapter assumes, of course, that international loan contracts are always respected, but we have not yet examined mechanisms that ensure compliance with their terms. We will study the question in Chapter 6.

### 1.1.4 Autarky Interest Rates and the Intertemporal Trade Pattern

Diagrams like Figure 1.1 can illuminate the main factors causing some countries to run initial current account deficits while others run surpluses. The key concept we need for this analysis is the autarky real interest rate, that is, the interest rate that would prevail in an economy barred from international borrowing and lending.

Were the economy restricted to consume at the autarky point $\mathbf{A}$ in Figure 1.1, the only real interest rate consistent with the Euler eq. (3) would be the autarky interest rate $r^{\mathrm{A}}$, defined by eq. (4) with outputs replacing consumptions:
$\frac{\beta u^{\prime}\left(Y_{2}\right)}{u^{\prime}\left(Y_{1}\right)}=\frac{1}{1+r^{\mathrm{A}}}$.
This equation also gives the autarky price of future consumption in terms of present consumption.
Figure 1.1 shows that when the latter autarky price is below the world relative price of future consumption-which is equivalent to $r^{A}$ being above $r$-future consumption is relatively cheap in the home economy and present consumption relatively expensive. Thus the home economy will "import" present consumption from abroad in the first period (by running a current account deficit) and "export" future consumption later (by repaying its foreign debt). This result is in accord
with the principle of comparative advantage from international trade theory, which states that countries tend to import those commodities whose autarky prices are high compared with world prices and export those whose autarky prices are comparatively low. ${ }^{7}$ It is the opportunity to exploit these pretrade international price differentials that explains the gains from trade shown in Figure 1.1.

A rise in present output or a fall in future output lowers the autarky real interest rate: either event would raise desired saving at the previous autarky interest rate, but since the residents of a closed endowment economy cannot save more in the aggregate without lending abroad, $r^{\mathrm{A}}$ must fall until people are content to consume their new endowment. Similarly, greater patience (a rise in $\beta$ ) lowers $r^{\mathrm{A}}$. By modifying Figure 1.1, you can check that when $r^{\mathrm{A}}$ is below the world interest rate $r$, the country runs a first-period current account surplus followed by a deficit, but still gains from trade.

It may come as a surprise that the existence of gains from intertemporal trade does not depend on the sign of the country's initial current account balance. The reason is simple, however. What produces gains is the chance to trade with someone different from oneself. Indeed, the greater is the difference, the greater the gain. The only case of no gain is the one in which, coincidentally, it happens that $r^{\mathrm{A}}=r$.

This reasoning also explains how changes in world interest rates affect a country's welfare. In Figure 1.1 the economy reaps trade gains by borrowing initially because its autarky interest rate is above the world rate, $r$. Notice, however, that, were the world interest rate even lower, the economy's welfare after trade would be higher than in Figure 1.1. The basic reason for this welfare gain is that a fall in the world interest rate accentuates the difference between the hoine country and the rest of the world, increasing the gains from trade. A small rise in the world interest rate (one that doesn't reverse the intertemporal trade pattern) therefore harms a first-period borrower but benefits a first-period lender.

### 1.1.5 Temporary versus Permanent Output Changes

A suggestive interpretation of the preceding ideas leads to a succinct description of how alternative paths for output affect the current account.

The natural benchmark for considering the effects of changing output is the case $\beta=1 /(1+r)$. The reason is that, in this case, eq. (7) becones
$\frac{u^{\prime}\left(Y_{2}\right)}{u^{\prime}\left(Y_{1}\right)}=\frac{1+r}{1+r^{A}}$,
which implies that the sole factor responsible for any difference between the world and autarky interest rates is a changing output level.

Imagine an economy that initially expects its output to be constant over time. The economy will plan on a balanced current account. But suppose $Y_{1}$ rises. If $Y_{2}$ does not change, the economy's autarky interest rate will fall below the world interest rate: a date 1 current account surplus will result as people smooth their consumption by lending some of their temporarily high output to foreigners. If $Y_{2}$ rises by the same amount as $Y_{1}$, however, the autarky interest rate does not change, and there is no current account imbalance. Alternatively, consumption automatically remains constant through time if people simply consume their higher output in both periods.

One way to interpret these results is as follows: permanent changes in output do not affect the current account when $\beta=1 /(1+r)$, whereas temporary changes do, temporary increases causing surpluses and temporary declines producing deficits. Likewise, a change in future expected output affects the sign of the current account in the same qualitative manner as an opposite movement in current output. We will generalize this reasoning to a many-period setting in the next chapter.

### 1.1.6 Adding Government Consumption

So far we have not discussed the role of a government. Government consumption is, however, easy to introduce.

Suppose government consumption per capita, $G$, enters the utility function additively, giving period utility the form $u(C)+v(G)$. This case is, admittedly, a simple one, but it suffices for the issues on which we focus. For now, it is easiest to suppose that the government simply appropriates $G_{t}$ in taxes from the private sector for $t=1,2$. This policy implies a balanced government budget each period (we will look at government deficits in Chapter 3). The representative private individual's lifetime budget constraint is thus
$C_{1}+\frac{C_{2}}{1+r}=Y_{1}-G_{1}+\frac{Y_{2}-G_{2}}{1+r}$.
Government spending also enters the date $t$ current account identity, which is now
$C A_{t}=B_{t+1}-B_{t}=Y_{t}+r_{t} B_{t}-C_{t}-G_{t}$.
The new feature here is that both government and private consumption are subtracted from national income to compute the current account. (Plainly we must account for all domestic expenditure-public as well as private-to reckon how much a country as a whole is saving.)

Since $G$ is beyond the private sector's control we can follow the same steps as in section 1.1.1 to conclude that the Euler equation (3) remains valid. Indeed, introducing government consumption as we have done here is equivalent
to relabeling the private sector's endowment $Y$ as output net of government con-
-
How do government consumption de in which $\beta=1 /(1+r)$, and output is conural benchmark once again is the case in consumption, private consumption would stant at $Y_{1}=Y_{2}=\bar{Y}$. Absent govern, with the current account balanced. Suppose, be constant in this case at $C=\bar{Y}$, with however, that $G_{1}>0$ while $G_{2}=0$. den of the temporary taxes to the future. The country therefore will run a deficit in period 1 and a surplus in period 2 . (5) implies that in the preceding example
Rep $\quad$ with $Y-G$ in eq. ( 5 ) $(1+r) G_{1}$
Government consumption in period 1 lowers private consumption, but by an overnment consumption in perioon is that the government consumption is temporary: it drops to zero in period 2. Thus the current account equation presented earlier in this subsection (recall that $B_{1}=0$ here) implies that

$$
C A_{1}=\bar{Y}-\bar{C}-G_{1}=-\frac{G_{1}}{2+r}<0
$$

In contrast, suppose that $G_{1}=G_{2}=\bar{G}$. Then consumption is constant at $\bar{C}=$ $\vec{Y}-\bar{G}$ in both periods, and the current account is balanced always. Government consumption affects the current account here only to the extent that it tilts the path of private net income.

* 1.1.7 A Digression on Intertemporal Preferences

Equation (1) assumes the representative individual's preferences are captured by a very particular lifetime utility function rather than an unrestricted function $U_{1}=$ $U\left(C_{1}, C_{2}\right)$. In eq. (1) consumption levels for different dates enter adith consumption over, the period utility function $u(C)$ is constant ovatural generalization of utility occurring over

$$
\begin{equation*}
U_{1}=\sum_{t=1}^{T} \beta^{t-1} u\left(C_{t}\right) . \tag{9}
\end{equation*}
$$

Preferences that can be represented by an additive lifetime utility function are called intertemporally additive preferences. The key property implied by intertemporal additivity is that the marginal rate of substitut for the preferences described
 by eq. (9)] is independent of consumption on any third date.
strictive (provided $T>2$, of course). It rules out certain kinds of intertemporal consumption dependencies, such as complementarity between total consumption levels in different periods. Such dependencies are at the heart of recent models of habit persistence in aggregate consumption. ${ }^{8}$

Although we will discuss particular alternative assumptions on tastes at several points in the book, the assumption of intertemporally additive preferences with an unvarying period utility function will form the backbone of our formal analysis. There are several reasons for this choice:

1. It is true that some types of goods, such as refrigerators and automobiles, are durable goods typically consumed over many periods rather than just one. This type of consumption linkage, however, is fundamentally technological. By defining utility over the flow of services from durables, and by imputing their rental cost, one can easily incorporate such goods within the umbrella of intertemporally additive preferences. (We show this in Chapter 2.)
2. For some types of goods, consumption at one point in time clearly does influence one's utility from consuming in closely neighboring periods. After eating a large meal, one is less inclined to want another an hour later. The time intervals of aggregation we look at in macroeconomic data, however, typically are measured in months, quarters, or years, periods over which many types of intertemporal dependencies fade.
3. Admittedly, even over long periods, habit persistence can be important. Drug addiction is an extreme example; watching television is a closely related one. In macroeconomics, however, one should think of preferences as being defined over consumption variables that really represent aggregate spending on a wide array of different goods. While we may have some intuition about the persistence effects of consuming certain items, it is harder to see obvious and quantitatively significant channels through which the totality of consumption has long-lived persistence effects.
4. One can think of some types of goods that most individuals would prefer to consume only once, such as marriage services. But even though consumption of such services is lumpy for an individual, it is relatively smooth in the aggregate.
5. If $G(\cdot)$ is any monotonically increasing function, then the utility function
$G\left[\sum_{r=1}^{T} \beta^{t-1} u\left(C_{t}\right)\right]$
naturally represents the same preferences as $U_{1}$ does, i.e., a monotonically increasing transformation of the lifetime utility function does not affect the consumer's underlying preference ordering over different consumption paths. Intertemporally additive preferences take the general form $G\left[u_{1}\left(C_{1}\right)+\cdots+u_{T}\left(C_{T}\right)\right]$ (with period utility functions possibly distinct). They also go by the name strongly intertemporally separable preferences. For further discussion of their implications, see Deaton and Muellbauer ( 1980 , ch. 5.3 ).

People get married all the time. Similarly, people may take vacation trips only at infrequent intervals, but this is not the case in the aggregate. (Seasonality can be important in either of these examples, but such effects are easily dealt with.)
5. Fundamentally, a very general intertemporally nonadditive utility function would yield few concrete behavioral predictions. If consumptions on different dates are substitutes, one gets dramatically different results from the case in which they are complements. Because maximal generality would lead to an unfalsifiable macroeconomic theory with little empirical content, macroeconomists have found it more fruitful to begin with a tractable basic setup like eq. (9), which has very sharp predictions. The basic setup can then be amended in parsimonious and testable ways if its implications seem counterintuitive or counterfactual.
6. In any event, while empirical research has raised interesting questions about the simplest time-additive preference model, it does not yet clearly point to a superior nonadditive alternative.

### 1.2 The Role of Investment

Historically, one of the main reasons countries have borrowed abroad is to finance productive investments that would have been hard to finance out of domestic savings alone. In the nineteenth century, the railroad companies that helped open up the Americas drew on European capital to pay laborers and obtain rails, rolling stock, and other inputs. To take a more recent example, Norway borrowed extensively in world capital markets to develop its North Sea oil resources in the 1970s after world oil prices shot up.

So far we have focused on consumption smoothing in our study of the current account, identifying the current account with national saving. In general, however, the current account equals saving minus investment. And because, in reality, investment usually is much more volatile than saving, to ignore investment is to miss much of the action.

### 1.2. Adding Investment to the Model

Let's modify our earlier model economy to allow for investment. We now assume that output is produced using capital, which, in tum, can be accumulated through investment. The production function for new output in either period is
$Y=F(K)$.
As usual production is strictly increasing in capital but subject to diminishing marginal productivity: $F^{\prime}(K)>0$ and $F^{\prime \prime}(K)<0$. Furthermore, output cannot be
produced without capital: $F(0)=0$. We will think of the representative consumer as having the additional role of producer with direct access to this technology. ${ }^{9}$

A unit of capital is created from a unit of the consumption good. This process is reversible, so that a unit of capital, after having been used to produce output, can be "eaten." You may find these assumptions unrealistic, but they help us sidestep some technical issues that aren't really central here. One key simplification due to our assumptions is that the relative price of capital goods in terms of consumption always equals 1 .

Introducing investment requires that we rethink the budget constraints individuals face, because now saving can flow into capital as well as foreign assets. Total domestic private wealth at the end of a period $t$ is now $B_{t+1}+K_{t+1}$, the sum of net foreign assets $B_{t+1}$ and the stock of domestic capital $K_{t+1} .{ }^{10}$

How is capital investment reflected in the date $t$ current account? The stock of capital $K_{t+1}$ accumulated through the end of period $t$ is the sum of preexisting capital $K_{t}$ and new investment during period $t, I_{t}$ (we ignore depreciation of capital):
$K_{t+1}=K_{t}+I_{t}$.
Nothing restricts investment to be nonnegative, so eq. (11) allows people to eat part of their capital.
Next, the change in total domestic wealth, national saving, is
$B_{t+1}+K_{t+1}-\left(B_{t}+K_{t}\right)=Y_{t}+r_{t} B_{t}-C_{t}-G_{t}$.
Finally, rearranging terms in this equation and substituting (11) shows that the current account surplus is
$C A_{t}=B_{t+1}-B_{t}=Y_{t}+r_{t} B_{t}-C_{t}-G_{t}-I_{t}$.
A very useful way to interpret the preceding current account identity is to label national saving as $S_{t}$ :
$S_{t} \equiv Y_{t}+r_{t} B_{t}-C_{t}-G_{t}$.
9. As we discuss in later chapters, it is reasonable to think of labor as being an additional production input alongside capital. A production function of the form (10) still is valid as long as labor is supplied inelastically by the individual producer. We assume
$\lim _{K \rightarrow 0} F^{\prime}(K)=\infty$
to ensure a positive capital stock.
10. It is simplest to suppose that all domestic capital is owned by domestic residents. The statement that total domestic wealth equals $B+K$ is true even when foreigners own part of the domestic capital stock, however, because domestic capital owned by foreigners is subtracted in calculating net foreign stock, however, because domestic capital owned by foreigners is subtracted in calculating net foreign
assets $B$. As long as perfect foresight holds, so that the ex post returns to assets are equal, the ownership assets $B$. As long as perfect foresight holds, so that the ex post returns to assets are equal, the ownership
of the domestic capital stock is irrelevant. The ownership pattern is not irrelevant, as we see later, when of the domestic capital stock is
unexpected shocks can occur.


Figure 1.2
Norway's saving-investment balance, 1973-94. (Source: OECD)
Then eq. (12) states that in an economy with investment,
$C A_{t}=S_{t}-I_{t}$.
National saving in excess of domestic capital formation flows into net foreign asset accumulation.

The saving-investment identity (14) discloses that the current account is fundamentally an intertemporal phenomenon. Simple as it is, the identity $C A=S-I$ is vital for analyzing how economic policies and disturbances change the current account. Will a protective tariff, often imposed to improve the current account, succeed in its aim? The answer cannot be determined from partial-equilibrium reasoning, but ultimately depends instead on how the tariff affects saving and investment.
Figure 1.2 returns to the Norwegian case mentioned at the start of this section, graphing recent data on saving, investment, and the current account. In the mid-1970s, the Norwegian current account registered huge deficits, touching -14 percent of GDP in 1977. In an accounting sense, higher energy-sector investment is "responsible" for much of the deficit, although saving simultaneously fell in the mid-1970s, possibly in anticipation of higher future oil revenues. Subsequent surpluses through 1985, reflecting higher saving and lower investment, enabled Norway to repay much of the debt incurred in the 1970s.
The Norwegian data illustrate an important point. The saving-investment identity is a vital analytical tool, but because $C A, S$, and $I$ are jointly determined endogenous variables that respond to common exogenous shocks, it may be mis-
leading to identify a specific ex post investment or saving shift as the "cause" of a current account change. Our model with investment will show how various exogenous shocks can simultaneously affect all three variables in the saving-investment identity.

### 1.2.2 Budget Constraint and Individual Maximization

To derive the intertemporal budget constraint analogous to eq. (8) when there is both government spending and investment, we simply add the asset-accumulation identities for periods 1 and 2. For period 1, current account eq. (12) gives
$B_{2}=Y_{1}-C_{1}-G_{1}-I_{1}$
(recall that $B_{1}=0$ ). For period 2, eq. (12) gives
$-B_{2}=Y_{2}+r B_{2}-C_{2}-G_{2}-I_{2}$
(recall that $B_{3}=0$ ). Solve this equation for $B_{2}$, and substitute the result into the equation that precedes it. One thereby arrives at the intertemporal budget constraint
$C_{1}+I_{1}+\frac{C_{2}+I_{2}}{1+r}=Y_{1}-G_{1}+\frac{Y_{2}-G_{2}}{1+r}$.
Now it is the present value of consumption plus investment that is limited by the present value of output.

In this economy with investment, a representative individual maximizes $U_{1}=$ $u\left(C_{1}\right)+\beta u\left(C_{2}\right)$ subject to eq. (15), where eq. (10) replaces $Y$ with $F(K)$ and eq. (11) is used to replace $I$ with the change in $K$. To simplify further, observe that people will never wish to carry capital past the terminal period 2 . Thus capital $K_{2}$ accumulated in period 1 will be consumed at the end of period 2 and $K_{3}$ will be zero, implying that
$I_{2}=K_{3}-K_{2}=0-K_{2}=-K_{2}$.
Using eq. (15) to eliminate $C_{2}$ from $U_{1}$ therefore transforms the individual's problem to

$$
\max _{C_{1}, I_{1}} u\left(C_{1}\right)
$$

$$
\begin{equation*}
+\beta u\left\{(1+r)\left[F\left(K_{1}\right)-C_{1}-G_{1}-I_{1}\right]+F\left(I_{1}+K_{1}\right)-G_{2}+I_{1}+K_{1}\right\} \tag{16}
\end{equation*}
$$

( $K_{1}$ is given by history and is not subject to choice on date 1.) The two corresponding first-order conditions are the Euler equation (3) and
$F^{\prime}\left(K_{2}\right)=r$,
where we have used the identity $K_{2}=K_{1}+I_{1}$.
An extra unit of output invested on date 1 can be fully consumed, together with its marginal contribution to output, $F^{\prime}\left(K_{2}\right)$, on date 2. Equation (17) says that

Box 1.1
Nominal versus Real Current Accounts

Our use in Figure 1.2 of data from official national income and product accounts raises an important measurement problem that you should recognize as you read this book. Unfortunately, the problem is easier to understand than to cure, so in most cases we reluctantly continue to rely on the official data.
Ideally, the current account should measure the change in an economy's net real claims on foreigners. In practice, however, government statistical agencies measure the current account and GDP by adding up the values of transactions measured in nominal terms, that is, in units of domestic money. This practice poses no conceptual hazards when money has a stable value in terms of real output, but, for reasons we will understand better after learning about monetary economics in Chapters $8-10$, real-world economies are almost always afflicted by at least some price-level inflation, a tendency for the money prices of all goods and services to rise over time. Such inflation would not be a problem if all international borrowing and lending involved the exchange of output-indexed bonds, as our theoretical model assumes. But most bonds traded between countries have returns and face values that are contracted in terms of currencies, implying that inflation can affect their real values.
A hypothetical example illustrates the problem. Suppose United States GDP is $\$ 7$ trillion dollars and the U.S. net foreign debt is $\$ 700$ billion. Suppose also that all international debts are linked to dollars, that the interest rate on dollar loans is 10 percent per year, and that U.S. GDP equals the sum of consumption, investment, and government spending. Under these assumptions the U.S. Department of Commerce would report the current account balance as the nominal interest outflow on net foreign assets, or $(0.1) \times(\$ 700$ billion $)=\$ 70$ billion. So measured, the current account
deficit is 1 percent of GDP.

Suppose, however, that all dollar prices are rising at 5 percent per year. Over the course of the year, the U.S. external debt declines in real value by $(0.05) \times(\$ 700$ billion $)=\$ 35$ billion as a result of this inflation. Thus the dollar value of the change in U.S. real net foreign assets is not $\$ 70$ billion, but $\$ 70$ billion $-\$ 35$ billion $=\$ 35$ billion. This smaller number divided by GDP, equal to 0.5 percent, shows the change in the economy's real net foreign assets as a fraction of real output. Naive use of nominal official numbers makes the deficit look twice as large relative to GDP as it really is!
While it was easy to measure the current account correctly in our example, doing so in practice is much harder. International financial transactions are denominated in many currencies. Changes in currency exchange rates as well as in national money price levels therefore enter into the real current account, but, because the currency composition of a country's net foreign debt is difficult to monitor, accurate correction is problematic. Many internationally held assets, such as stocks, long-term bonds, and real estate, can fluctuate sharply in value. Accounting for these price changes involves
similar problems.
Caveat emptor. Unless otherwise stated, the ratios of the current account to output that you encounter in this book are the rough approximations one gleans from official national accounts. The same is true of related wealth flows, such as saving-to-output
ratios.
period 1 investment should continue to the point at which its marginal return is the same as that on a foreign loan. A critical feature of eq. (17) is its implication that the desired capital stock is independent of domestic consumption preferences! Other things equal, wouldn't a less patient country, one with a lower value of $\beta$, wish to invest less? Not necessarily, if it has access to perfect international capital markets. A country that can borrow abroad at the interest rate $r$ never wishes to pass up investment opportunities that offer a net rate of return above $r$.
Several key assumptions underpin the separation of investment from consumption decisions in this economy. First, the economy is small. The saving decisions of its residents don't change the interest rate at which investment projects can be financed in the world capital market. Second, the economy produces and consumes a single tradable good. When the economy produces nontraded goods, as in some of Chapter 4's models, consumption shifts can affect investment. Third, capital markets are free of imperfections that might act to limit borrowing. We shall see later (in Chapter 6) that when factors such as default risk restrict access to international borrowing, national saving can influence domestic investment. ${ }^{11}$

In the present setup, investment is independent of government consumption as well. In particular, government consumption does not crowd out investment in a small open economy facing a perfect world capital market.

### 1.2.3 Production Possibilities and Equilibrium

Let's assume temporarily that government consumption is zero in both periods. Then Figure 1.3 shows how the current account is determined when there is investment. To the information in Figure 1.1, Figure 1.3 adds an intertemporal production possibilities frontier (PPF) showing the technological possibilities available in autarky for transforming period 1 consumption into period 2 consumption. The PPF is described by the equation
$C_{2}=F\left[K_{1}+F\left(K_{1}\right)-C_{1}\right]+K_{1}+F\left(K_{1}\right)-C_{1}$.
What does this equation imply about the PPF's position and shape? If the economy chose the lowest possible investment level on date 1 by eating all its inherited capital immediately (setting $I_{1}=-K_{1}$ ), it would enjoy the highest date 1 consumption available in autarky, $C_{1}=K_{1}+F\left(K_{1}\right)$. In this case date 2 consumption
11. Once we allow for uncertainty, as in Chapters 2 and 5 , restrictions on the tradability of certain assets also can upset the separation of investment from consumption. We have not yet introduced labor as an explicit factor of production, but if the supply of labor influences the marginal product of capital, the separation can also fail when consumption and labor effort enter the period utility function in a nonadditive manner.
Even if consumption shifts don't alter investment, the converse proposition need not be true! As budget constraint (15) shows, investment enters the consumer's budget constraint in equilibrium, so in general factors that shift domestic investment can affect national saving too.


Figure 1.3
Investment and the current account
would be at its lowest possible level, $C_{2}=F(0)+0=0$. The resulting point is the PPF's horizontal intercept in Figure 1.3. At the other extreme, the economy could invest all output on date 1 and eat no inherited capital $K_{1}$. This decision would set $C_{1}=0, I_{1}=F\left(K_{1}\right), K_{2}=K_{1}+F\left(K_{1}\right)$, and $C_{2}=F\left[K_{1}+F\left(K_{1}\right)\right]+$ $K_{1}+F\left(K_{1}\right)$, the last being the highest date 2 consumption available in autarky. The PPF's vertical intercept in Figure 1.3 shows the allocation just described. In between, the PPF's slope is obtained from (18) by differentiation:
$\frac{\mathrm{d} C_{2}}{\mathrm{~d} C_{1}}=-\left[1+F^{\prime}\left(K_{2}\right)\right]$.
Capital's diminishing marginal productivity makes the PPF strictly concave, as shown. ${ }^{12}$
Point $\mathbf{A}$ in Figure 1.3 is the autarky equilibrium. There, the PPF is tangent to the highest indifference curve the economy can reach without trade. The common slope of the two curves at $\mathbf{A}$ is $-\left(1+r^{\mathbf{A}}\right)$, where $r^{\mathrm{A}}$, as earlier, is the autarky real interest rate. All three closed-economy equilibrium conditions hold at poin A. First, producer maximization: investment decisions are efficient, given $r^{\wedge}$ [that

[^2]is, condition (17) holds for $\left.r^{\mathrm{A}}\right]$. Second, consumer maximization: the intertemporal Euler condition holds, again given $r^{A}$. Third, output-market clearing: consumption and investment sum to output in both periods. You can see that markets clear at A by observing that in autarky $K_{2}$ equals the distance between $C_{1}$ and $Y_{1}+K_{1}$ along the horizontal axis, so that $C_{1}+I_{1}=Y_{1}$, whereas eq. (18) can be written as $C_{2}=Y_{2}+K_{2}=Y_{2}-I_{2}$.

In Figure 1.3, the economy faces a world interest rate, $r$, lower than the autarky rate $r^{A}$ implied by the dual tangency at point $\mathbf{A}$. Thus, at $\mathbf{A}$, the marginal domestic investment project offers a rate of return above the world cost of borrowing. The opportunity to trade across periods with foreigners lets domestic residents gain by investing more and producing at point $\mathbf{B}$, through which the economy's new budget line passes. Production at $\mathbf{B}$ maximizes the present value of domestic output (net of investment) by placing the economy on the highest feasible budget line at world prices. Consuming at point $\mathbf{C}$ gives the economy the highest utility level it can afford.

The horizontal distance between points $\mathbf{A}$ and $\mathbf{B}$ is the extra investment generated by opening the economy to the world capital market. The horizontal distance between points $\mathbf{A}$ and $\mathbf{C}$ shows the extra first-period consumption that trade simultaneously allows. Since total first-period resources $Y_{1}+K_{1}$ haven't changed, the sum of these two horizontal distances--the distance from $\mathbf{B}$ to $\mathbf{C}$-is the firstperiod current account deficit.

The utility curve through point $\mathbf{C}$ lies above the one through point $\mathbf{A}$. The distance between them measures the gains from trade. In Figure 1.1, trade gains were entirely due to smoothing the time path of consumption. In Figure 1.3 there is an additional source of gain, the change in the economy's production point from $\mathbf{A}$ to $B$.

Had the world interest rate $r$ been above $r^{A}$ rather than below it, the country would have run a first-period current account surplus but still enjoyed gains from intertemporal trade, as in the pure endowment case studied earlier.

### 1.2.4 The Model with Government Consumption

In section 1.2 .3 we assumed away government consumption. Now we reinstate it in our graphical analysis.

A glance at eqs. (15) (for the individual's intertemporal budget constraint) and (18) (for the PPF) shows how changes in government consumption affect the graphs of these two relations between $C_{2}$ and $C_{1}$ : both are shifted vertically downward by the amount of the increase in $G_{2}$ and horizontally leftward by the amount of the increase in $G_{1}$.

In understanding the difference that government consumption can make, it helps intuition to begin with an economy in which government consumption is always zero and the current account is in balance, so that consumption and production are


Figure 1.4
Government consumption and the current account
at point $\mathbf{A}$ in Figure 1.4. Compare this economy with an otherwise identical one in which $G_{1}>0$ while $G_{2}$ remains at zero. In the second economy, both the PPF and the budget constraint have shifted to the left by an amount $G_{1}$, and the economy's production point is $\mathbf{B}$, implying the same investment level as at $\mathbf{A} .{ }^{13}$ Notice, however, that as long as consumption is a normal good on both dates, consumers will not wish to consume at $\mathbf{B}$, for this choice would imply an unchanged $C_{2}$. Instead, they respond to a lower lifetime income level by reducing consumption on both dates and choosing consumption point $\mathbf{C}$, which is southeast of $\mathbf{B}$ on the new budget constraint.
We conclude from Figure 1.4 that, other things equal, an economy with disproportionately high period 1 government consumption will have a current account deficit in that period. When government consumption is expected to occur on the future date 2 instead, the current account will be in surplus on date 1 . Both predictions are explained by individuals' efforts to spread the burden of higher taxes over both periods of life through borrowing or lending abroad.

[^3]
### 1.3 A Two-Region World Economy

Until now we have focused on a country too small to affect the world interest rate. In this section we show how the world interest rate is determined and how economic events in large regions are transmitted abroad.

### 1.3.1 A Global Endowment Economy

Let us start by abstracting from investment again and assuming a world of two regions or countries, called Home and Foreign, that receive exogenously determined endowments on dates 1 and 2 . The two economies have parallel structures, but symbols pertaining to Foreign alone are marked by asterisks. We also omit government spending, which operates precisely as a reduction in output in our model.
Equilibrium in the global output market requires equal supply and demand on each date $t=1,2$,
$Y_{t}+Y_{t}^{*}=C_{t}+C_{t}^{*}$.
Equivalently, subtracting world consumption from both sides in this equation implies world saving must be zero for $t=1,2$,
$S_{t}+S_{t}^{*}=0$.
Since there is so far no investment in the model, this equilibrium condition is the same as $C A_{t}+C A_{t}^{*}=0$. We can simplify further by recalling that when there are only two markets, output today and output in the future, we need only check that one of them clears to verify general equilibrium (Walras's law). Thus the world economy is in equilibrium if
$S_{1}+S_{1}^{*}=0$.
Figure 1.5 shows how the equilibrium world interest rate is determined for given present and future endowments. In this case a country's date 1 saving depends only on the interest rate it faces. Curve SS shows how Home saving depends on $r$ and curve $\mathbf{S}^{*} \mathbf{S}^{*}$ does the same for Foreign. We will probe more deeply into the shapes of the saving schedules in a moment, but for now we ask you to accept them as drawn in Figure 1.5.

In Figure 1.5, the equilibrium world interest rate makes Home's lending, measured by the length of segment AB, equal to Foreign's borrowing, measured by the length of $\mathbf{B}^{*} \mathbf{A}^{*}$. The equilibrium world interest rate $r$ must lie between the two autarky rates:
$r^{A}<r<r^{A *}$.


Figure 1.5
Global exchange equilibrium

In Home, the rise in the interest rate from its autarky level encourages saving, leading to positive Home saving of AB. Home's first-period current account surplus also equals AB. Foreign's situation is the reverse of Home's, and it runs a current account deficit of $\mathbf{B}^{*} \mathbf{A}^{*}$. (Because Home and Foreign face the same world interest rate, we do not mark it with an asterisk.) The intertemporal trade pattern naturally conforms to the comparative advantage principle.
It is easy to see in Figure 1.5 how changes in exogenous variables alter the world interest rate and international capital flows. A ceteris paribus increase in Home's date 1 output, as we know, leads the country to raise its saving at a given rate of interest. As a result, SS shifts to the right. Plainly, the new equilibrium calls for a lower world interest rate, higher Home lending on date 1, and higher Foreign borrowing. Other things equal, higher date 2 output in Home shifts SS leftward, with opposite effects. Changes in Foreign's intertemporal endowment pattern work similarly, but through a shift of the Foreign saving schedule $\mathbf{S}^{*} \mathbf{S}^{*}$.
An important normative issue concerns the international distribution of the benefits of economic growth. Is a country helped or hurt by an increase in trading partners' growth rates? To be concrete, suppose Home's date 2 output $Y_{2}$ rises, so that $\mathbf{S S}$ shifts leftward and the world interest rate rises. Because Foreign finds that the terms on which it must borrow have worsened, Foreign is actually worse off. Home, conversely, benefits from a higher interest rate for the same reason: the terms on which it lends to Foreign have improved. Thus, alongside the primary gain due to future higher output, Home enjoys a secondary gain due to the induced change in the intertemporal relative price it faces.

An interest-rate increase improves the intertemporal terms of trade of Home, which is "exporting" present consumption (through a date 1 surplus), while worsening those of Foreign, which is "importing" present consumption (through a date 1 deficit). In general, a country's terms of trade are defined as the price of its exports in terms of its imports. Here, $1+r$ is the price of present consumption in terms of future consumption, that is, the price of a date 1 surplus country's export good in terms of its import good. As in static trade theory, a country derives a positive welfare benefit when its terms of trade improve and a negative one when they worsen.

It may seem reasonable to suppose that if Home's date 1 output $Y_{1}$ rises Home must benefit. In this case, however, the last paragraph's reasoning works in reverse. Because the world interest rate falls, Home's terms of trade worsen and counteract the primary benefit to Home of higher date 1 output. (Part of Home's benefit is exported abroad, and Foreign's welfare unambiguously rises.) Indeed, the terms-of-trade effect can be so big that higher date 1 output for Home actually worsens its lot. This paradoxical outcome has been dubbed immiserizing growth by Bhagwati (1958).

## Application: War and the Current Account

Nothing in human experience is more terrible than the misery and destruction caused by wars. Their high costs notwithstanding, wars do offer a benefit for empirical economists. Because wars have drastic consequences for the economies involved, usually are known in advance to be temporary, and, arguably, are exogenous, they provide excellent "natural experiments" for testing economic theories

Wartime data can have drawbacks as well. During wars, market modes of allocation may be supplemented or replaced by central economic planning. Because price controls and rationing are common, data on prices and quantities become hard to interpret in terms of market models. Matters are even worse when it comes to testing open-economy models, as wars inevitably bring tighter government control over capital movements and trade. Sometimes the normal data collecting and processing activities of statistical agencies are disrupted.

One way to reduce some of these problems is to focus on data from before the 1930s, when governments decisively turned away from laissez-faire in attempts to shield their economies from the worldwide Great Depression. Although pre-1930s data can be of uneven quality compared with modern-day numbers, they have been surprisingly useful in evaluating modern theories. We illustrate the use of historical data by looking briefly at the effects on both bystanders and participants of some early twentieth-century wars.

Sweden did not directly participate in World War I, while Japan took part only peripherally. Current-account data for the war's 1914-18 span are available


Figure 1.6
Current accounts of Japan and Sweden, 1861-1942
for both countries. What does our model predict about the effect of a foreign war on nonbelligerents? Return to Figure 1.5, interpreting "Home" as the warring portion of the world. For inhabitants of Home the war represents a situation in which the output available for private consumption has exogenously become much lower in the present than in the future. In response, Home lowers its saving at every interest rate, causing SS to shift to the left. Home's current account surplus falls (and may become a deficit), and the world interest rate rises. In Foreign, the region still at peace, the rise in the world interest rate causes a rise in saving and an improved current account balance (perhaps even a surplus).

Figure 1.6, which graphs current account data for Japan and Sweden, is consistent with the prediction that nonparticipants should run surpluses during large foreign wars. In both countries there is an abrupt shift from secular deficit toward a massive surplus reaching 10 percent of national product. The huge surpluses disappear once the war is over.

What is the evidence that belligerents do wish to borrow abroad? Foreign financing of wars has a long history; over the centuries, it has helped shape the institutions and instruments of international finance. From the late seventeenth century through the end of the Napoleonic Wars, lenders in several other continental countries underwrote Britain's military operations abroad. As far back as the first half of the fourteenth century, Edward III of England invaded France with the help of

Table 1.2
Japan's Gross Saving and Investment During the Russo-Japanese War (fraction of GDP)

| Year | Saving/GDP | Investment/GDP |
| :--- | :--- | :--- |
| 1903 | 0.131 | 0.136 |
| 1904 | 0.074 | 0.120 |
| 1905 | 0.058 | 0.168 |
| 1906 | 0.153 | 0.164 |

loans from Italian bankers. Edward's poor results in France and subsequent refusal to honor his foreign debts illustrate a potential problem for tests of the hypothesis that wars worsen the current account. Even though a country at war may wish to borrow, why should lenders respond when a country's ability to repay may be impaired even in the event of victory? The prospect that borrowers default can limit international capital flows, as we discuss in greater detail in Chapter 6. But intergovernment credits often are extended in wartime, and private lenders may stay in the game, too, if the interest rates offered them are high enough to compensate for the risk of loss.

Japan's 1904-1905 conflict with Russia offers a classic example of large-scale borrowing to finance a war. In February 1904, tensions over Russia's military presence in Manchuria and its growing influence in Korea erupted into open hostilities. Public opinion on the whole favored Japan, but Russia's superiority in manpower and other resources led more sober commentators to predict that the great power would beat its upstart challenger in the long run. These predictions quickly faded as Japan's naval prowess led to a string of victories that helped lay bare the fragility of tsarist Russia's social, political, and economic fabric.

The Russian surrender of Port Arthur in January 1905 decisively gave Japan the upper hand. Over the war's course Japan's government borrowed tens of millions of pounds sterling in London, New York, and Berlin. In 1904, Japan had to pay an interest rate of around $7 \frac{1}{2}$ percent per year on its borrowing, but by 1905, with the war's ultimate outcome no longer in doubt, lenders were charging Japan only around $5 \frac{1}{2}$ percent. The Japanese neutralized Russia's naval forces in June 1905, and peace was concluded the following September.
The Russo-Japanese War offers an unusually good testing ground for the model we have developed: it caused no disruption of global financial markets and there was a fair amount of certainty as to the eventual winner. Figure 1.6 shows that Japan's current account moved sharply into deficit during the war, with foreign borrowing topping 10 percent of GDP in 1905. Also consistent with the our model, national saving dropped sharply in the years 1904 and 1905, as shown in Table 1.2.

### 1.3.2 Saving and the Interest Rate

We now justify the shapes of the saving schedules drawn in Figure 1.5. This reasoning requires an understanding of the complex ways a change in the interest rate affects the lifetime consumption allocation.

### 1.3.2.1 The Elasticity of Intertemporal Substitution

The key concept elucidating the effects of interest rates on consumption and saving is the elasticity of intertemporal substitution, which measures the sensitivity of the intertemporal consumption allocation to an interest-rate change.
To see the role of intertemporal substitutability in determining the demands for consumption on different dates, take natural logarithms of the across-date firstorder condition (4) and compute the total differential

$$
\begin{align*}
\mathrm{d} \log (1+r) & =\frac{u^{\prime \prime}\left(C_{1}\right)}{u^{\prime}\left(C_{1}\right)} \mathrm{d} C_{1}-\frac{u^{\prime \prime}\left(C_{2}\right)}{u^{\prime}\left(C_{2}\right)} \mathrm{d} C_{2} \\
& =\frac{C_{1} u^{\prime \prime}\left(C_{1}\right)}{u^{\prime}\left(C_{1}\right)} \mathrm{d} \log C_{1}-\frac{C_{2} u^{\prime \prime}\left(C_{2}\right)}{u^{\prime}\left(C_{2}\right)} \mathrm{d} \log C_{2} . \tag{20}
\end{align*}
$$

Define the inverse of the elasticity of marginal utility by
$\sigma(C)=-\frac{u^{\prime}(C)}{C u^{\prime \prime}(C)}$.
The parameter defined in eq. (21) is called the elasticity of intertemporal substitution. When $\sigma$ is constant, eq. (20) becomes
$\mathrm{d} \log \left(\frac{C_{2}}{C_{1}}\right)=\sigma \mathrm{d} \log (1+r)$.
Intuitively, a gently curving period utility function (a high $\sigma$ ) implies a sensitive relative consumption response to an interest-rate change.

The class of period utility functions characterized by a constant elasticity of intertemporal substitution is
$u(C)=\frac{C^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}, \quad \sigma>0$.
We refer to this class of utility functions as the isoelastic class. For $\sigma=1$, the righthand side of eq. (22) is replaced by its limit, $\log (C) .{ }^{14}$

[^4]$u(C)=\frac{C^{1-\frac{1}{\sigma}}-1}{1-\frac{1}{\sigma}}$

### 1.3.2.2 The Shape of the Saving Schedule

To determine the date 1 consumption response to an interest-rate change, use Home's intertemporal budget constraint, $C_{2}=(1+r)\left(Y_{1}-C_{1}\right)+Y_{2}$, to eliminate $C_{2}$ from its Euler equation, $u^{\prime}\left(C_{1}\right)=(1+r) \beta u^{\prime}\left(C_{2}\right)$. (We are assuming $B_{1}=0$.) The result is
$u^{\prime}\left(C_{1}\right)=(1+r) \beta u^{\prime}\left[(1+r)\left(Y_{1}-C_{1}\right)+Y_{2}\right]$.
Implicitly differentiating with respect to $r$ gives
$\frac{\mathrm{d} C_{1}}{\mathrm{~d} r}=\frac{\beta u^{\prime}\left(C_{2}\right)+\beta(1+r) u^{\prime \prime}\left(C_{2}\right)\left(Y_{1}-C_{1}\right)}{u^{\prime \prime}\left(C_{1}\right)+\beta(1+r)^{2} u^{\prime \prime}\left(C_{2}\right)}$.
Let's assume for simplicity that $u(C)$ is isoelastic with constant intertemporal substitution elasticity $\sigma$. We can then divide the numerator and denominator of the last equation by $u^{\prime}\left(C_{2}\right) / C_{2}$ and, using definition (21) and the Euler equation (3), express the derivative as
$\frac{\mathrm{d} C_{1}}{\mathrm{~d} r}=\frac{\left(Y_{1}-C_{1}\right)-\sigma C_{2} /(1+r)}{1+r+\left(C_{2} / C_{1}\right)}$.
The numerator shows that a rise in $r$ has an ambiguous effect on Home's date 1 consumption. The negative term proportional to $\sigma$ represents substitution away from date 1 consumption that is entirely due to the rise in its relative price. But there is a second term, $Y_{1}-C_{1}$, that captures the terms-of-trade effect on welfare of the interest rate change. If Home is a first-period borrower, $C_{1}>Y_{1}$, the rise in the interest rate is a terms-of-trade deterioration that makes it poorer. As eq. (24) shows, this effect reinforces the pure relative-price effect in depressing $C_{1}$. But as $r$ rises and Home switches from borrower to lender, the terms-of-trade effect reverses direction and begins to have a positive influence on $C_{1}$. For high enough interest rates, $C_{1}$ could even become an increasing function of $r$. If $Y_{1}-C_{1}>0$, we can be sure that $\mathrm{d} C_{1} / \mathrm{d} r<0$ only if $r$ is not too far from the Home autarky rate.

Since date 1 output is given at $Y_{1}$, these results translate directly into conclusions about the response of saving $S_{1}$, which equals $Y_{1}-C_{1}$. The result is a saving schedule $\mathbf{S S}$ such as the one in Figure 1.5. (Of course, the same principles govern an analysis of Foreign, from which the shape of $\mathbf{S} * \mathbf{S} *$ follows.)
if we want it to converge to logarithmic as $\sigma \rightarrow 1$. To see convergence, we now can use L'Hospital's rule. As $\sigma \rightarrow 1$, the numerator and denominator of the function both approach 0 . Therefore, we can differentiate both with respect to $\sigma$ and get the answer by taking the limit of the derivatives' ratio, $C^{1-\frac{1}{\sigma}} \log (C)$, as $\sigma \rightarrow 1$.
Subtracting the constant $1 /\left(1-\frac{1}{\sigma}\right)$ from the period utility function does not alter economic behavior: the utility function in eq. (22) has exactly the same implications as the alternative function. To avoid burdening the notation, we will always write the isoelastic class as in eq. (22), leaving it implicit that one must subtract the appropriate constant to derive the $\sigma=1$ case.


[^0]:    4. The Swiss mathematician Leonhard Euler (1707-1783) served at one time as the court mathematician to Catherine the Great of Russia. The dynamic equation bearing his name arose originally in the problem of finding the so-called brachistochrone, which is the least-time path in a vertical plane for an object pulled by gravity between two specified points.
[^1]:    5. One does not need to assume literally that all individuals are identical to conclude that aggregate consumption will behave as if chosen by a single maximizing agent. Under well-defined but rather stringent preference assumptions, individual behavior can be aggregated exactly, as discussed by Deaton and Muellbauer (1980, ch. 6). We defer a formal discussion of aggregation until Chapter 5. For a perspective on ways in which the representative-agent paradigm can be misleading, however, see Kirman (1992).
[^2]:    12. To test your understanding, show that the second derivative $\mathrm{d}^{2} C_{2} / \mathrm{d} C_{1}^{2}=F^{\prime \prime}\left(K_{2}\right)<0$.
[^3]:    13. It may seem odd at first glance that in autarky a rise in $G_{1}$ alone reduces the maximal output available for private consumption on date 2 as well as on date 1 . Recall, however, that when private consumption is zero on date 1 , investment is lower by $G_{1}$ in autarky, so the maximal date 2 consumption available to the autarkic economy is only $F\left(K_{1}+Y_{1}-G_{1}\right)+K_{1}+Y_{1}-G_{1}$.
[^4]:    14. We really have to write the isoelastic utility function as
