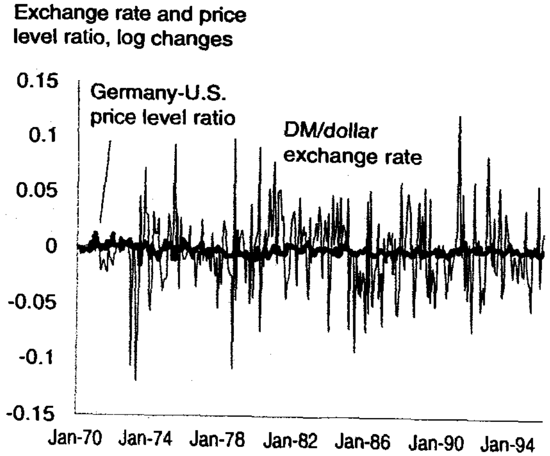

Figure 9.2
Germany and the United States, exchange rate and price changes

At the microeconomic level, an impact of exchange-rate volatility on the intercountry relative prices of similar commodities has been extensively documented. Many studies find that deviations from the law of one price are highly correlated with nominal exchange-rate changes.[3] Engel (1993) compares U.S. with Canadian consumer price data for a large variety of goods including fuel, men's clothing, and apples. In more than 2,000 pairwise comparisons, Engel finds that with only a few exceptions, the relative prices of *similar* goods across the United States and Canada are more volatile than the relative prices of *dissimilar* goods within either country. These findings are reinforced in Engel and Rogers (1995), who extend the comparisons to 23 American and Canadian cities. Even after controlling for distance between two cities, they find an enormous "border effect" on volatility.[4] For example, the volatility of relative prices for very similar consumer goods appears to be much greater between closely neighboring American and Canadian city pairs such as Buffalo and Toronto or Seattle and Vancouver, than between cities such as New York and Los Angeles, which lie on opposite sides of the North American continent but within the same country.

This evidence motivates a look at a classic sticky-price extension of the flexible-price monetary model of Chapter 8.

issue remains controversial. We will argue in section 9.3.3 that the choice of exchange-rate regime had dramatic consequences for the international transmission of the Great Depression of the 1930s.

3. See, for example, Isard (1977) and Giovannini (1988).

4. For surveys of evidence on deviations from the law of one price, see Froot and Rogoff (1995) and Rogoff (1996).

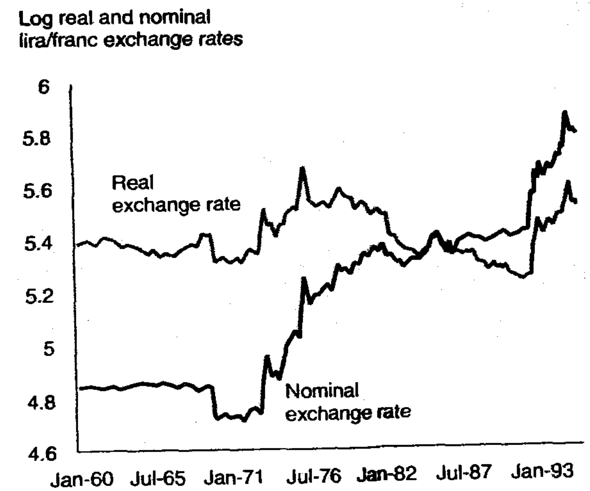

Figure 9.3
France and Italy, exchange rate and prices (liras/franc, natural logarithms)

9.2 The Mundell-Fleming-Dornbusch Model

Since the early 1960s, the dominant policy paradigm for studying open-economy monetary and fiscal policy issues has been the Keynesian framework developed by Mundell (1963, 1964) and Fleming (1962). This section presents a variant of Dornbusch's (1976) famous perfect-foresight extension of the Mundell-Fleming model. As we have already noted, Dornbusch's model has important deficiencies, including its inability to deal adequately with current-account and fiscal-policy dynamics or, more fundamentally, with welfare issues. Nevertheless, the model remains so influential as to warrant discussion in any serious treatment of international monetary theory. Nothing here is essential for understanding the more complete models of Chapter 10. The Dornbusch setup will, however, help the reader frame many of the basic questions in international monetary economics and give perspective on some of the newer developments in the area.

9.2.1 A Small-Open-Economy Model with Sticky Prices and Endogenous Output

The Dornbusch model includes some of the same building blocks as the Cagan-type monetary models we discussed in Chapter 8. A small country faces an exogenous world (foreign-currency) interest rate i^*, which is assumed constant. With open capital markets and perfect foresight, uncovered interest parity must hold:

$$i_{t+1} = i^* + e_{t+1} - e_t. \tag{1}$$

[As in Chapter 8, $\mathsf{i}_{t+1} = \log(1 + i_{t+1})$ is the logarithm of the gross domestic nominal interest rate between periods t and $t+1$, $\mathsf{i}^* = \log(1 + i^*)$, and e is the logarithm of the exchange rate, defined as the domestic price of foreign currency.][5] As in the basic monetary model of Chapter 8, only domestic residents hold the domestic money, and domestic monetary equilibrium is characterized by the Cagan-type aggregate relationship

$$\mathsf{m}_t - \mathsf{p}_t = -\eta \mathsf{i}_{t+1} + \phi \mathsf{y}_t, \tag{2}$$

where m is the log of the nominal money supply, p is the log of the domestic-currency price level, and y is the log of domestic output.

Let p^* be the (log of the) foreign price level measured in foreign currency. The model assumes that purchasing power parity (PPP) need not hold, so that the (log) real exchange rate, $\mathsf{e} + \mathsf{p}^* - \mathsf{p} \equiv \log(\mathcal{E}P^*/P)$, can vary. Here we take the domestic consumption basket as numeraire and define the real exchange rate as the relative price of the foreign consumption basket. (See Chapter 4 for a discussion of real exchange rates. As in that chapter, we refer to a rise in $\mathcal{E}P^*$ relative to P as a home real depreciation or, alternatively, as a real depreciation of the home currency. A fall in $\mathcal{E}P^*$ relative to P is a real appreciation for the home country.)

The Dornbusch model effectively aggregates all domestic output as a single composite commodity and assumes that aggregate demand for home-country output, y^d, is an increasing function of the home real exchange rate $\mathsf{e} + \mathsf{p}^* - \mathsf{p}$:

$$\mathsf{y}_t^d = \bar{\mathsf{y}} + \delta(\mathsf{e}_t + \mathsf{p}^* - \mathsf{p}_t - \bar{\mathsf{q}}), \qquad \delta > 0. \tag{3}$$

(We hold p^* constant throughout.) Interpret the constant $\bar{\mathsf{y}}$ in eq. (3) as the "natural" rate of output. If we denote the real exchange rate by

$$\mathsf{q} \equiv \mathsf{e} + \mathsf{p}^* - \mathsf{p}, \tag{4}$$

then we can interpret $\bar{\mathsf{q}}$ in eq. (3) as the *equilibrium* real exchange rate consistent with full employment. For simplicity, we usually assume both $\bar{\mathsf{y}}$ and $\bar{\mathsf{q}}$ to be constant.

The assumption in eq. (3) that a rise in the foreign price level relative to that in the home country (a rise in $\mathsf{e} + \mathsf{p}^*$ relative to p) shifts world demand toward home-produced goods could be justified through several mechanisms. Mundell, Fleming, and Dornbusch assume that the home country has monopoly power over the tradables it produces (despite its smallness in asset markets) and that home-produced tradables have a greater CPI weight at home than abroad. Real depreciation might

5. When Mundell and Fleming wrote, macroeconomists had not yet applied methods for handling rational expectations. In their principal models, Mundell and Fleming basically assumed static exchange rate expectations by requiring that $\mathsf{i}_t = \mathsf{i}^*$ under perfect capital mobility. Equation (1) plays a central role in Dornbusch's (1976) extension of the Mundell-Fleming model.

also increase demand for home goods by shifting domestic spending from foreign tradables to domestic nontradables.

Positing an aggregate demand function such as eq. (3) without deriving it from underlying microfoundations constitutes a sharp methodological departure from the approach we have generally adopted in most other parts of this book. However, like the Cagan and Solow models of earlier chapters, the Dornbusch model yields some important insights that survive more careful derivation. Chapter 10 will model the demand side in more detail.

Although asset markets clear at every moment as in the models of Chapter 8, output markets need not in the Dornbusch model. If goods prices were fully flexible, as in the models of Chapter 8, output would always equal its natural level, so that $\mathsf{y}_t^d = \mathsf{y}_t = \bar{\mathsf{y}}$, and thus q would always equal $\bar{\mathsf{q}}$. As we have already seen, the assumption of flexible prices is quite unrealistic. In practice, nominal goods prices adjust much more slowly than exchange rates. In the Dornbusch model, the empirical reality of sticky prices is captured by assuming that p is *predetermined*, and responds only slowly to shocks.

If the price level cannot move immediately to clear markets, however, then unanticipated shocks plainly can lead to excess demand or supply. In the absence of market clearing, one must make some kind of assumption about how the actual level of output is determined. Here we will follow Keynesian tradition and simply assume that output is demand determined, so that $\mathsf{y}_t = \mathsf{y}_t^d$. For the moment, we are not able to offer any justification for this assumption, and we will leave all details of aggregate supply in the background. Fortunately, it will be possible to give a much more satisfactory treatment once we have introduced microfoundations for aggregate supply in Chapter 10.[6]

Although p_t is predetermined and cannot respond instantly to date t shocks, it does adjust slowly over time in response to excess demand. Specifically, the price level adjusts according to the inflation-expectations-augmented Phillips curve

$$\mathsf{p}_{t+1} - \mathsf{p}_t = \psi(\mathsf{y}_t^d - \bar{\mathsf{y}}) + (\tilde{\mathsf{p}}_{t+1} - \tilde{\mathsf{p}}_t), \tag{5}$$

where

$$\tilde{\mathsf{p}}_t \equiv \mathsf{e}_t + \mathsf{p}_t^* - \bar{\mathsf{q}}_t$$

is the price level that would prevail if the output market cleared (*given* e_t, p_t^*, and $\bar{\mathsf{q}}_t$). Intuitively, the first term on the right-hand side of eq. (5) embodies the price inflation caused by date t excess demand, while the second term provides for the price-level adjustment needed to keep up with *expected* inflation or productivity

6. A model with implications very similar to those of the Dornbusch model introduces a labor market and assumes that it is the nominal wage, rather than the price level, that is predetermined. (See, for example, Obstfeld, 1985, or Rogoff, 1985a.) Qualitatively similar results also follow from a variant of the model in which some prices are temporarily rigid while others are fully flexible.

[As in Chapter 8, $i_{t+1} = \log(1 + i_{t+1})$ is the logarithm of the gross domestic nominal interest rate between periods t and $t+1$, $i^* = \log(1 + i^*)$, and e is the logarithm of the exchange rate, defined as the domestic price of foreign currency.][5] As in the basic monetary model of Chapter 8, only domestic residents hold the domestic money, and domestic monetary equilibrium is characterized by the Cagan-type aggregate relationship

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$$p_{t+1} - p_t = \psi(y_t^d - \bar{y}) + (\tilde{p}_{t+1} - \tilde{p}_t), \tag{5}$$

where

$$\tilde{p}_t \equiv e_t + p_t^* - \bar{q}_t$$

is the price level that would prevail if the output market cleared (*given* e_t, p_t^*, and $\bar{q}_t$). Intuitively, the first term on the right-hand side of eq. (5) embodies the price inflation caused by date t excess demand, while the second term provides for the price-level adjustment needed to keep up with *expected* inflation or productivity

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growth. That is, the second term captures the movement in prices that would be needed to keep $y = \bar{y}$ if the output market were in equilibrium.[7] Differencing the definition of $\bar{p}_t$ gives

$$\bar{p}_{t+1} - \bar{p}_t = (e_{t+1} + p^*_{t+1} - \bar{q}_{t+1}) - (e_t + p^*_t - \bar{q}_t).$$

Substituting this expression into eq. (5), and recalling that p^* and $\bar{q}$ are assumed constant, we find

$$p_{t+1} - p_t = \psi(y^d_t - \bar{y}) + e_{t+1} - e_t. \tag{6}$$

This completes our description of the Dornbusch model.

9.2.2 Graphical Solution of the Dornbusch Model

To solve the model, we begin by using eqs. (3) and (4) to express eq. (6) as

$$\Delta q_{t+1} = q_{t+1} - q_t = -\psi\delta(q_t - \bar{q}). \tag{7}$$

We will assume $1 > \psi\delta$, which ensures that shocks to the real exchange rate damp out monotonically over time. Next, we substitute eqs. (1), (3), and (4) into eq. (2). Together with the simplifying normalizations $p^* = \bar{y} = i^* = 0$, this step yields

$$m_t - e_t + q_t = -\eta(e_{t+1} - e_t) + \phi\delta(q_t - \bar{q}) \tag{8}$$

or

$$\Delta e_{t+1} = e_{t+1} - e_t = \frac{e_t}{\eta} - \frac{(1 - \phi\delta)q_t}{\eta} - \left(\frac{\phi\delta\bar{q} + m_t}{\eta}\right). \tag{9}$$

Equations (7) and (9) constitute a system of two first-order difference equations in q and e. It is not difficult to solve them analytically (see Supplement C to Chapter 2), but it is instructive to consider first the simple phase diagram in Figure 9.4, which is drawn under the assumption that m_t is constant at $\bar{m}$. Under eq. (7), the $\Delta q = 0$ schedule is vertical at $q = \bar{q}$. Thus the speed of anticipated real adjustment is independent of nominal factors. The $\Delta e = 0$ schedule has vertical-axis intercept $\phi\delta\bar{q} + \bar{m}$, and it is upward-sloping as drawn provided $1 > \phi\delta$, a condition we provisionally assume. (Note that the slope must be below 45 degrees.) The steady-state pair $(\bar{q}, \bar{e})$ lies at the intersection of the two curves. It follows from eq. (8) that

$$\bar{e} = \bar{m} + \bar{q}, \tag{10}$$

which, using the definition of q [eq. (4)], implies $\bar{p} = \bar{m}$ (recall that $p^* = 0$).[8]

7. There are several ways to allow for price-level adjustment in the Dornbusch model, and eq. (5) is based on Mussa (1982). The original Dornbusch (1976) model was designed only to analyze one-time shocks, and is not general enough to allow for anticipated disturbances; see Frankel (1979), Mussa (1982, 1984), Obstfeld and Rogoff (1984), and Obstfeld and Stockman (1985).

8. The solution for $\bar{e}$ follows because in the steady state, $q = \bar{q}$, $m = \bar{m}$, and $e_{t+1} = e_t$.

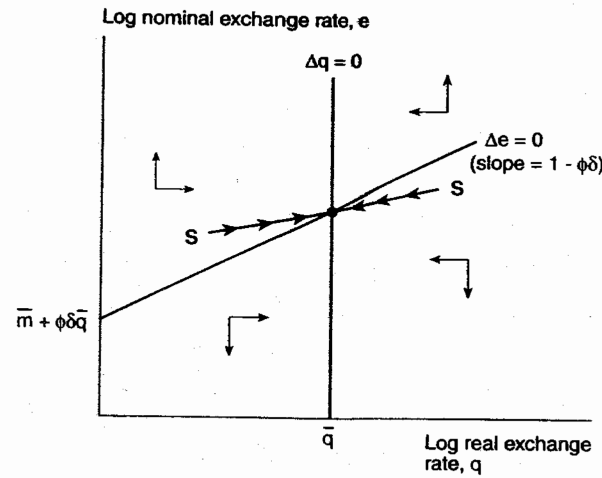

Figure 9.4
The Dornbusch model

Figure 9.4 also describes the system's dynamics away from the steady state. The dynamic arrows show that the Dornbusch model has the saddle-path property familiar from earlier chapters. That is, along all paths beginning anywhere but along the upward-sloping saddle path, **SS**, the exchange rate will eventually implode or explode. Since we did not derive the money demand function (2) from microfoundations, there is no way to argue rigorously that the economy is constrained to be on SS. One can, however, appeal to the close analogy between this model and the maximizing models of Chapter 8, and to the fact that the no-bubbles path is the only one that tightly links prices to fundamentals.

Now let's consider the time-honored thought experiment of an unanticipated permanent rise in the money supply from $\bar{m}$ to $\bar{m}'$. In the long run, of course, both the exchange rate e and the price level p must increase in proportion to the change in the money supply,

$$\bar{p}' - \bar{p} = \bar{e}' - \bar{e} = \bar{m}' - \bar{m},$$

as is easily shown using eqs. (7) and (9); the long-run interest rate remains i^*. In the short run, however, the price level p is predetermined and cannot respond to the unanticipated money change. Assuming that the economy initially occupies the steady-state equilibrium corresponding to $m_t = \bar{m}$ for all t, then, on initial date 0,

$$p_0 = \bar{m}, \tag{11}$$

which implies that

$$q_0 = e_0 - \bar{m}. \tag{12}$$

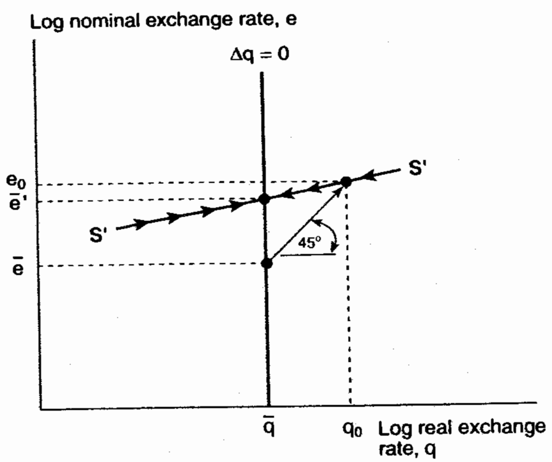

Figure 9.5
Exchange rate overshooting

The economy's immediate response to the unanticipated money shock and its transition to the new steady state are graphed in Figure 9.5. The new saddle path $\mathbf{S'S'}$ passes through the new steady state $(\bar{q}, \bar{m}' + \bar{q})$. The 45° arrow in the figure is the initial condition described by eq. (12), which has a slope of unity. (Note that the slope of the saddle path $\mathbf{S'S'}$ must be less than unity since it is shallower than the $\Delta e = 0$ schedule, which itself has slope less than one.) Thus, in response to the unanticipated money supply increase, the economy initially jumps to point (q_0, e_0) at the intersection of the new (post-shock) saddle path and the initial condition (12). Note the $e_0 > \bar{e}'$. That is, the exchange rate initially changes *more than proportionately* to the money shock.

This is Dornbusch's celebrated "overshooting" result. One reason it captured the imagination of many international economists and policymakers was its implication that the surprising volatility of floating exchange rates might be consistent with rational expectations. The 1970s were years of monetary instability throughout the industrialized world. If volatile money supplies had amplified effects on exchange rates, Dornbusch reasoned, they might be substantially responsible for the sharp exchange-rate fluctuations observed after the onset of floating in 1973.

The intuition underlying Dornbusch's overshooting result is easily seen by referring to the money demand equation (2), rewritten here:

$$m_t - p_t = -\eta i_{t+1} + \phi y_t.$$

The increase in m causes an increase in real money balances of $\bar{m}' - \bar{m}$, since p is initially fixed. Suppose the exchange rate jumped immediately to its new steady state. Then eq. (3) implies that output would rise (on impact) by $\delta(\bar{m}' - \bar{m})$;

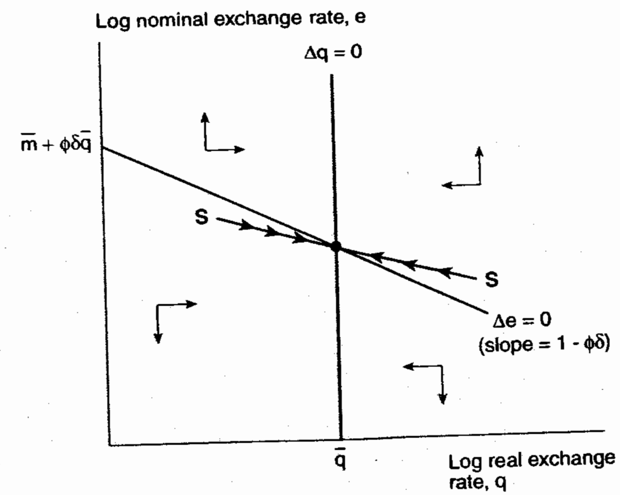

Figure 9.6
Exchange rate undershooting

money demand would thus rise by $\phi\delta(\bar{m}' - \bar{m})$. If, as we assumed in drawing Figures 9.4 and 9.5, $\phi\delta < 1$, then the rise in money demand is less than the rise in money supply. Thus, the home nominal interest rate i must fall below i^* to restore money-market equilibrium. This conclusion contradicts our initial supposition that e jumps immediately to its new steady state, for $i < i^*$ implies an expected *fall* in e, by interest parity equation (1). What does happen? By the preceding logic, the exchange rate cannot rise in the short run by *less* than $\bar{m}' - \bar{m}$. There would again have to be a fall in i, and therefore an expected appreciation—an impossibility, for then the exchange rate would be traveling away from rather than toward its steady state. The only possibility, when $\phi\delta < 1$, is for the currency initially to *overshoot* its long-run level. In the short-run equilibrium i still falls, but future appreciation is a rational expectation if the initial exchange rate e_0 is above its eventual steady state $\bar{e}'$.[9]

The preceding discussion suggests that overshooting will not necessarily occur if output responds sharply to exchange rate depreciation (δ is large) and if the income elasticity of money demand, ϕ, is large. Figure 9.6 graphs the case $\phi\delta > 1$, in which the $\Delta e = 0$ schedule is downward sloping. The saddle path $\mathbf{SS}$ now has a negative slope, as the arrows of motion show. An unanticipated permanent rise in the money supply therefore makes the exchange rate e rise less than proportionately to the money-supply increase. (The postshock exchange

9. Uncovered interest parity is an ex ante relationship that need not hold ex post if there are unanticipated shocks. Since we are allowing for an unanticipated shock at time 0, uncovered interest parity places no constraint on the initial movement in e_0.

rate e_0 lies below the new long-run rate $\bar{e}'$.) Notice that, regardless of whether the exchange rate undershoots or overshoots, the dynamics of the real exchange rate and output are qualitatively the same. The nominal depreciation of domestic currency implies a real depreciation (since prices are sticky). This real depreciation raises aggregate demand, so output rises temporarily above its steady-state value $\bar{y}$.

9.2.3 Analytical Solution of the Dornbusch Model

We have illustrated the main ideas of the Dornbusch model graphically. For completeness, however, we now show the model's analytical solution. Our approach to solving the model exploits its recursive structure.

Given any date t deviation of the real exchange rate from its long-run value, the solution to eq. (7) [rewritten as $q_{t+1} - \bar{q} = (1 - \psi\delta)(q_t - \bar{q})$] is

$$q_s - \bar{q} = (1 - \psi\delta)^{s-t}(q_t - \bar{q}), \qquad s \geq t. \tag{13}$$

(Recall that we have assumed $1 - \psi\delta > 0$.)[10] Having solved for the path of the real exchange rate (as a function of q_t), we can derive the path of the nominal exchange rate e with relative ease. For an exogenously given path of q, eq. (9) can be viewed as a first-order difference equation virtually identical to the Cagan model of Chapter 8, and it is solved similarly. Solving eq. (9) for e_t, and then subtracting $\bar{q}$ from both sides, yields

$$e_t - \bar{q} = \frac{\eta}{1+\eta}(e_{t+1} - \bar{q}) + \frac{1-\phi\delta}{1+\eta}(q_t - \bar{q}) + \frac{m_t}{1+\eta}.$$

By iterative forward substitution for $e_s - \bar{q}$, one obtains

$$e_t - \bar{q} = \frac{1}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t} m_s + \frac{1-\phi\delta}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t}(q_s - \bar{q}) \tag{14}$$

after eliminating speculative bubbles by imposing the condition

$$\lim_{T\to\infty}\left(\frac{\eta}{1+\eta}\right)^T e_{t+T} = 0.$$

If the money supply is constant at $\bar{m}$ as is assumed in the figures, eq. (14) reduces to

$$e_t - \bar{q} = \bar{m} + \frac{1-\phi\delta}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t}(q_s - \bar{q}). \tag{15}$$

To evaluate eq. (15), we substitute for $q_s - \bar{q}$ using eq. (13) to get

10. See Supplement C to Chapter 2.

$$e_t - \bar{q} = \bar{m} + \frac{1-\phi\delta}{1+\eta}(q_t - \bar{q})\sum_{s=t}^{\infty}(1-\psi\delta)^{s-t}\left(\frac{\eta}{1+\eta}\right)^{s-t},$$

which simplifies to the equation for the saddle path SS,

$$e_t = \bar{m} + \bar{q} + \frac{1-\phi\delta}{1+\psi\delta\eta}(q_t - \bar{q}). \tag{16}$$

Notice that we constrained the economy to lie on the saddle path by imposing the no-speculative-bubbles condition following eq. (14). Also note that the slope of the saddle path depends on $1 - \phi\delta$, as demonstrated in the earlier diagrammatic analysis.

We can now solve analytically for the initial jumps in the real and nominal exchange rate that occur if the economy is at a steady state on initial date 0 when an unanticipated permanent increase in the money supply from $\bar{m}$ to $\bar{m}'$ occurs. The (postshock) date 0 real exchange rate, q_0, is found by combining the initial condition (12) (which embodies the assumption that $p_0 = \bar{m}$ is predetermined) together with saddle-path equation (16) (putting $\bar{m}'$ in place of $\bar{m}$, and setting $t = 0$). The second equilibrium condition embodies the assumption that the economy jumps immediately to the new, *postshock*, saddle path. The result is

$$q_0 = \bar{q} + \frac{1+\psi\delta\eta}{\phi\delta+\psi\delta\eta}(\bar{m}' - \bar{m}).$$

Since, by eq. (12), $e_0 = q_0 + \bar{m}$,

$$e_0 = \bar{m} + \bar{q} + \frac{1+\psi\delta\eta}{\phi\delta+\psi\delta\eta}(\bar{m}' - \bar{m}). \tag{17}$$

We see that the nominal exchange rate overshoots its new long-run equilibrium if $1 > \phi\delta$. Finally, to obtain the nominal exchange rate's transition path leading to the new long-run equilibrium, we combine the equation preceding eq. (17) with eqs. (13) and (16) to obtain

$$e_t = \bar{m}' + \bar{q} + (1-\psi\delta)^t\left[\frac{1-\phi\delta}{\phi\delta+\psi\delta\eta}(\bar{m}' - \bar{m})\right].$$

Can *real* shocks also lead to overshooting? Suppose that at time 0, there is an unanticipated fall in $\bar{q}$ to $\bar{q}'$. What is the adjustment process? The answer, as one can most easily deduce by comparing the steady-state relationship (10) and the initial condition (12), is that the domestic currency appreciates immediately to its new long-run level, $\bar{m} + \bar{q}'$, and the economy immediately goes to the new steady state.[11] The reason is that the required adjustment in the *real* exchange rate can be

11. That is, eqs. (7) and (9) will continue to hold with $\Delta q_{t+1} = \Delta e_{t+1} = 0$ if e_{t+1}, e_t, q_t, and $\bar{q}$ all change by the same amount $\bar{q}' - \bar{q}$.

accommodated in equilibrium entirely by a change in the *nominal* rate. It therefore does not necessitate any change in the long-run price level.

9.2.4 More General Money-Supply Processes

Our analysis has focused on one-time permanent increases in the money supply, but it is straightforward to extend the model to allow for temporary money-supply increases. Indeed, we have already done most of the work needed to solve the general case. Using eq. (13) once again to simplify the second summation term in eq. (14) yields

$$e_t - \bar{q} = \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1+\eta}\right)^{s-t} m_s + \frac{1-\phi\delta}{1+\psi\delta\eta}(q_t - \bar{q})$$

or

$$e_t - e_t^{flex} = \frac{1-\phi\delta}{1+\psi\delta\eta}(q_t - \bar{q}), \tag{18}$$

where

$$e_t^{flex} \equiv \bar{q} + \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1+\eta}\right)^{s-t} m_s = \bar{q} + p_t^{flex}. \tag{19}$$

One can interpret e_t^{flex} and p_t^{flex} as the exchange rate and price level that would obtain if output prices were perfectly flexible (in which case q_t would equal $\bar{q}$).[12]

Instead of thinking of a one-time unanticipated change in the money supply, consider a date 0 change in the (perhaps very general) money supply process that (unexpectedly) changes e_t^{flex} and p_t^{flex} to $(e_t^{flex})'$ and $(p_t^{flex})'$ respectively, where $(e_t^{flex})' - e_t^{flex} = (p_t^{flex})' - p_t^{flex}$, since money shocks don't affect the real exchange rate when prices are flexible. We assume $p_0 = p_0^{flex}$. Then it is straightforward to use eq. (18) together with a generalized version of the initial condition (12),

$$q_0 = e_0 - p_0^{flex},$$

and eq. (19) to obtain

$$e_0 - e_0^{flex} = \frac{1+\psi\delta\eta}{\phi\delta+\psi\delta\eta}[(e_0^{flex})' - e_0^{flex}]. \tag{20}$$

[Compare eq. (20) with eq. (17) for the case of a permanent increase in the money supply, remembering that e_0^{flex} changes one-for-one with $\bar{q}$.] Finally, eqs. (13), (18), and (20) imply

$$e_t - (e_t^{flex})' = (1-\psi\delta)^t[e_0 - (e_0^{flex})']. \tag{21}$$

Assuming that $1 > \phi\delta$, eq. (20) implies that any date 0 disturbance that causes an unanticipated rise in e_0^{flex} will cause an even larger unanticipated rise in e_0. With very general money supply processes, it is no longer meaningful to talk about overshooting with respect to a fixed long-run equilibrium nominal exchange rate. But one can say that the *impact* exchange-rate effect of a monetary shock is greater when prices are sticky than when they are flexible. Thus price stickiness affects the conditional variance of the exchange rate. Equation (21) says the exchange rate converges to its (moving) flexible-price equilibrium value after a shock at a rate given by $\psi\delta$.

So far, our entire analysis has been for the perfect-foresight case, augmented by one-time unanticipated shocks. Because the model is (log) linear, however, it is straightforward to generalize it to the case where the money supply is explicitly stochastic, much along the lines of the log-linear models we considered in earlier chapters.[13] We leave this as an exercise.

9.2.5 Money Shocks, Nominal Interest Rates, and Real Interest Rates

Perhaps the most important insight gained by introducing more general money-supply processes concerns the different patterns of exchange-rate correlation with real and nominal interest rates.[14] In the simplest version of the Dornbusch model, in which there are only permanent unanticipated changes in the level of the money supply, lower nominal interest rates on a currency are associated with depreciation. In the flexible-price models of Chapter 8, however, we found that shocks to the *growth rate* of the money supply lead to the opposite correlation: increases in the nominal interest rate are associated with currency depreciation. When there is a money-supply growth-rate shock, which effect dominates?

Unlike money-supply-level shocks, growth-rate shocks lead to a positive correlation between nominal interest rates and exchange rates in the Dornbusch model. Suppose that the money supply is initially governed by the process

$$m_t = \bar{m} + \mu t$$

and that the economy is in a steady state (i.e., has converged to the long-run flexible-price equilibrium). The nominal interest rate in this steady state must be $i^* + \mu$. Then, at time 0, there is an unanticipated rise in the expected *future* money growth rate from μ to μ' so that

12. Notice that $\bar{p}_t$ defined just after eq. (5), differs from p_t^{flex} in being the hypothetical price level that would clear the output market at the *current* (possibly disequilibrium) nominal exchange rate, not, as in eq. (19), at the flexible-price nominal exchange rate e_t^{flex}. Indeed p_t^{flex} is precisely the equilibrium price level we derived in the flexible-price Cagan model [see eq. (9) in Chapter 8].

13. See, for example, Mussa (1982, 1984).

14. Frankel (1979), in a classic paper, stressed the importance of distinguishing between real and nominal interest rates in empirical exchange-rate modeling. Frankel's paper was the first serious effort to implement the Dornbusch model empirically.