Population

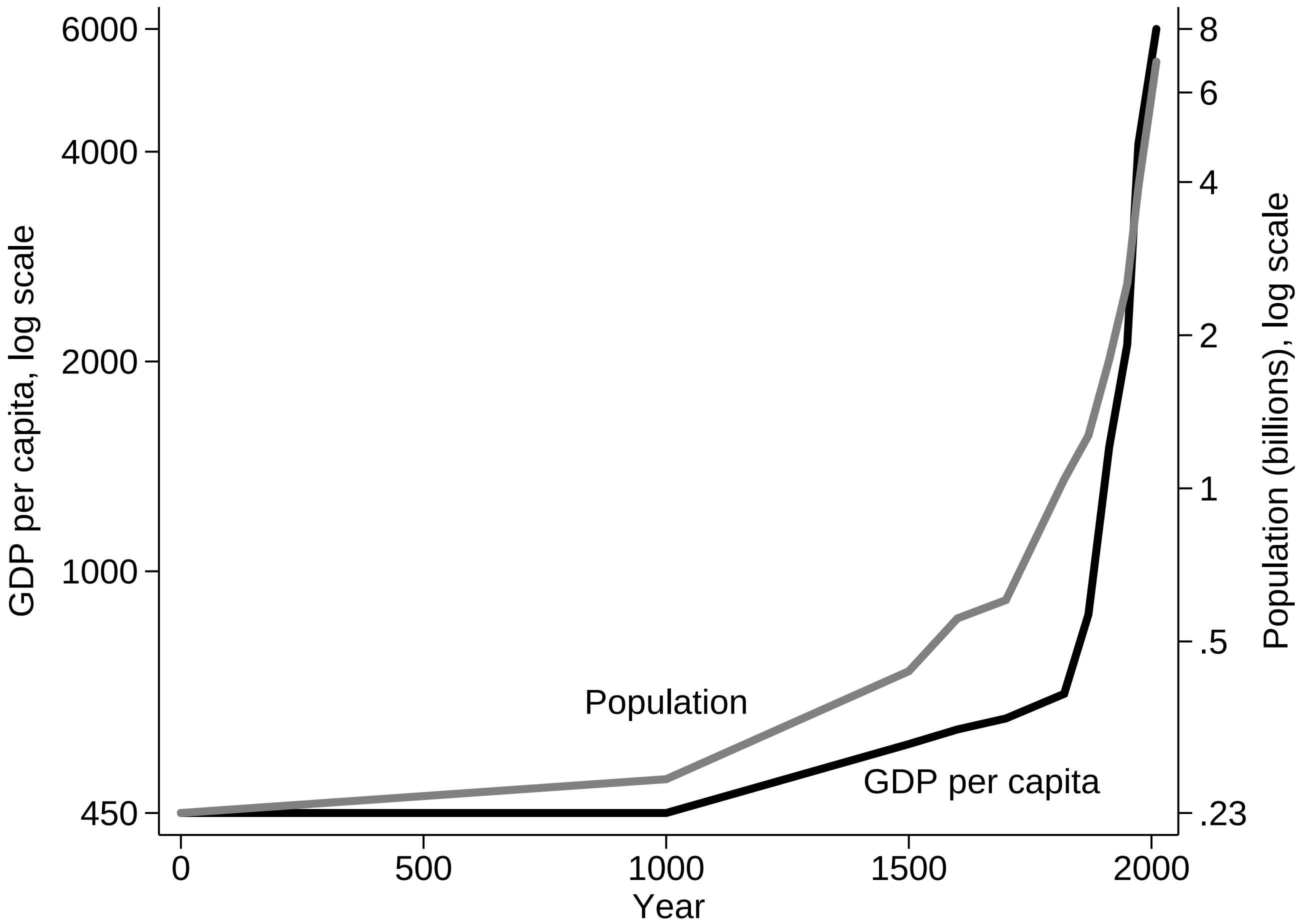
We've seen conflicting effects of population:

- Negative in the Solow model: more people means spreading capital across more workers
- Positive in the Romer/Schumpeter model: more people means more ideas/innovations

Can add another possible negative effect. *Malthusian* effects occur when there is a fixed/limited resource (e.g. land) and then more people lowers living standards.

While Malthusian effects appear to be true, overall the positive effect of population wins over the long run.

Population and Output per worker



Eras

- Malthusian Era (1 million BC to 1800-ish AD)
  - Low population growth rates: 0.02-0.27% per year
  - Low y growth rates: 0-0.14% per year
- Post-Malthusian Era (1800-ish AD to 1920-ish AD)
- Both population and y accelerate
- Population growth rates: 0.4-1.0% per year
- y growth rates: 0.5-1.3% per year
- Modern Growth Era (1920-ish AD to now)
  - Continued y growth, but Demographic Transition
  - Population growth rates: falling below 0.5% per year
  - y growth rates: approaching long-run trend of 1.85% per year

Malthusian Economy

Production now includes a fixed factor, X

$$Y = BX^{\beta}L^{1-\beta} \tag{1}$$

where B is a measure of productivity. Output per worker is

$$y = B\left(\frac{X}{L}\right)^{\beta} \tag{2}$$

Note that output per worker depends negatively on L - this is the Malthusian effect

Population growth is now endogenous - determined inside the model

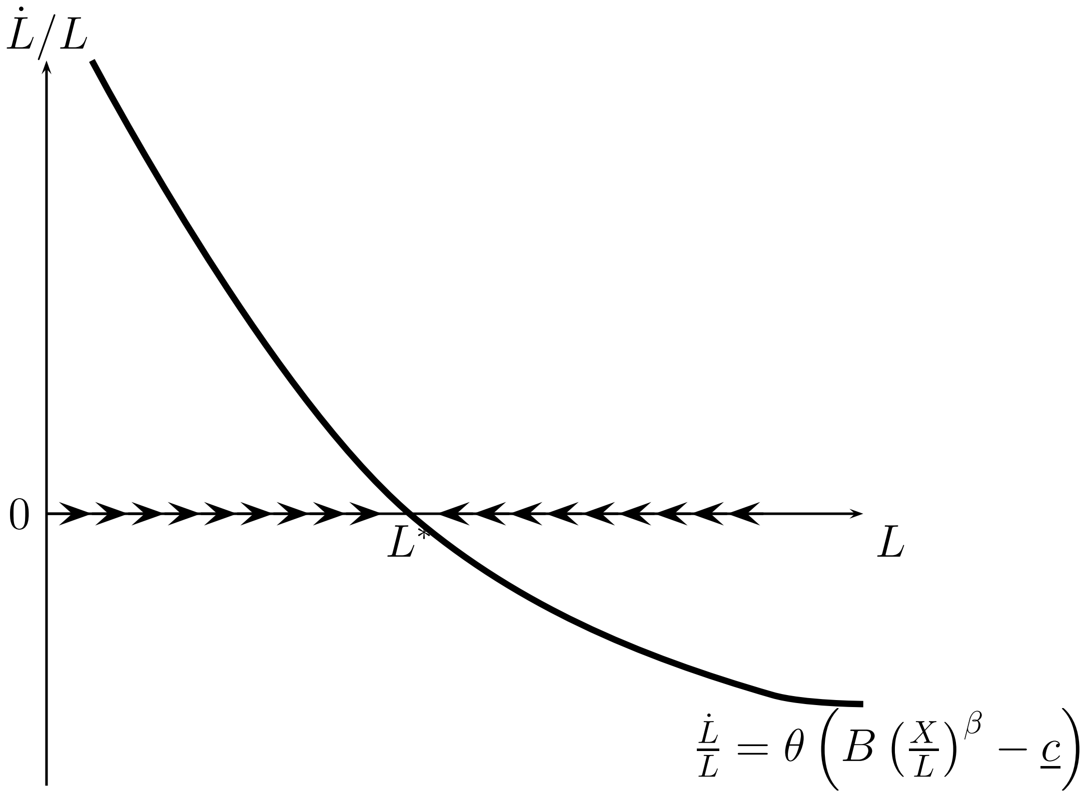
$$\frac{\dot{L}}{L} = \theta(y - \underline{c}) \tag{3}$$

where $\underline{c}$ is the subsistence constraint. Below that, population falls as people do not consume enough (i.e. famine) while above $\underline{c}$ population rises.

Malthusian Equilibrium

Combine the two equations to get

$$\frac{\dot{L}}{L} = \theta\left(\left(\frac{X}{L}\right)^{\beta} - \underline{c}\right) \tag{4}$$



Malthusian Equilibrium

In steady state, we have that

$$L^* = \left(\frac{B}{\underline{c}}\right)^{1/\beta} X \tag{5}$$

and

$$y^* = \underline{c}. \tag{6}$$

Our model gives us a stagnant output per worker. Any productivity increase (B) gets translated into higher populations, not higher living standards.

$\underline{c}$ need not be bare miniumum subsistence, but it is the fixe output per worker in a Malthusian economy

Dismal Science

Implications of the Malthusian model

- Anything that kills lots of people (L drops) will raise living standards
- Eventually people will breed themselves back to lower living standards

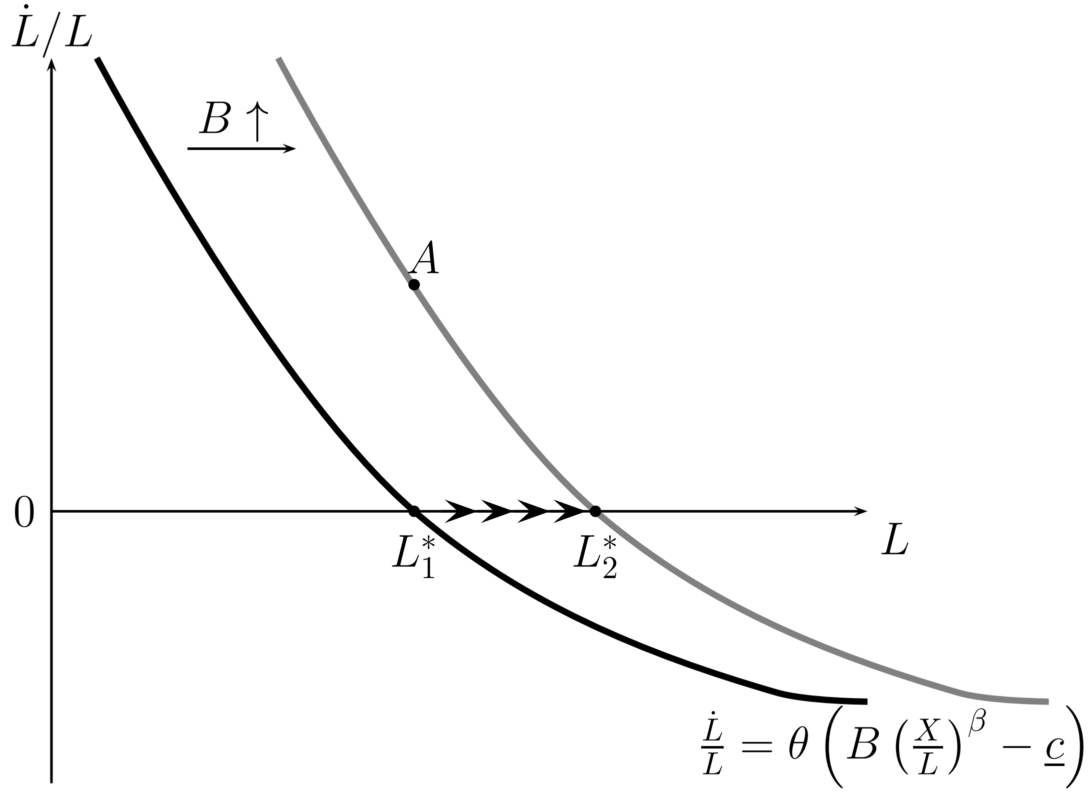
Black Death in Europe, 14th-15th century

- England: population falls from 3.75 million to 2 million, real wage doubles
- Italy: population falls from 10 million to 7 million, real wage up by 2.5 times

By 1500 both had wages back to pre-Black Death levels, and populations back at prior levels

Technological Change

A one-off shift up in B raises steady-state population



Continuous Technological Change

Take the logs and derivative of production function

$$y = B\left(\frac{X}{L}\right)^{\beta} \tag{7}$$

to get

$$\frac{\dot{y}}{y} = \frac{\dot{B}}{B} - \frac{1}{\beta}\frac{\dot{L}}{L} \tag{8}$$

and let $g = 1/\beta\dot{B}/B$ for convenience.

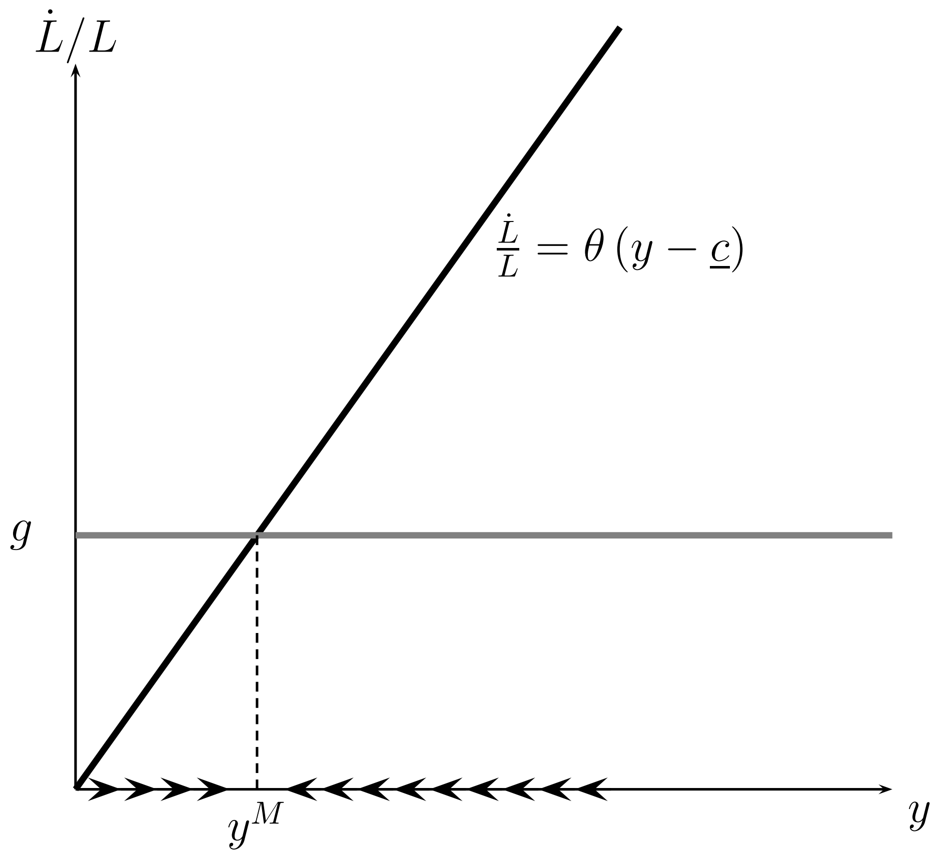
Then

- If $\dot{L}/L > g$, output per worker is rising
- If $\dot{L}/L < g$, output per worker is falling
- If $L\dot{/}L = g$, output per worker is constant

With Population Growth

Combine this with

$$\frac{\dot{L}}{L} = \theta(y - \underline{c}) \qquad (9)$$



Endogenous Technology

With constant technological progress at g

- Population size grows continuously (matching Malthusian era data)
- Steady state output per worker is stagnant (matching Malthusian era data)

We match history better. But where does g come from? We can introduce endogenous technological progress

$$\frac{\dot{B}}{B} = \nu \frac{s_R L^{\lambda}}{B^{1-\phi}} \tag{10}$$

which is just like our endogenous model from before. But focus on non-steady state behavior.

If L is very small compared to B, then as L goes up, $\dot{B}/B$ goes up. So the more people, the faster is technological progress. Scale effects.

Kremer Model

Simplify endogenous technology by setting $\lambda = 1$ and $\phi = 1$. (We said $\phi = 1$ is wrong, generally, but will work for the period of time when L was very low). Assume $s_R = 1$ for simplicity.

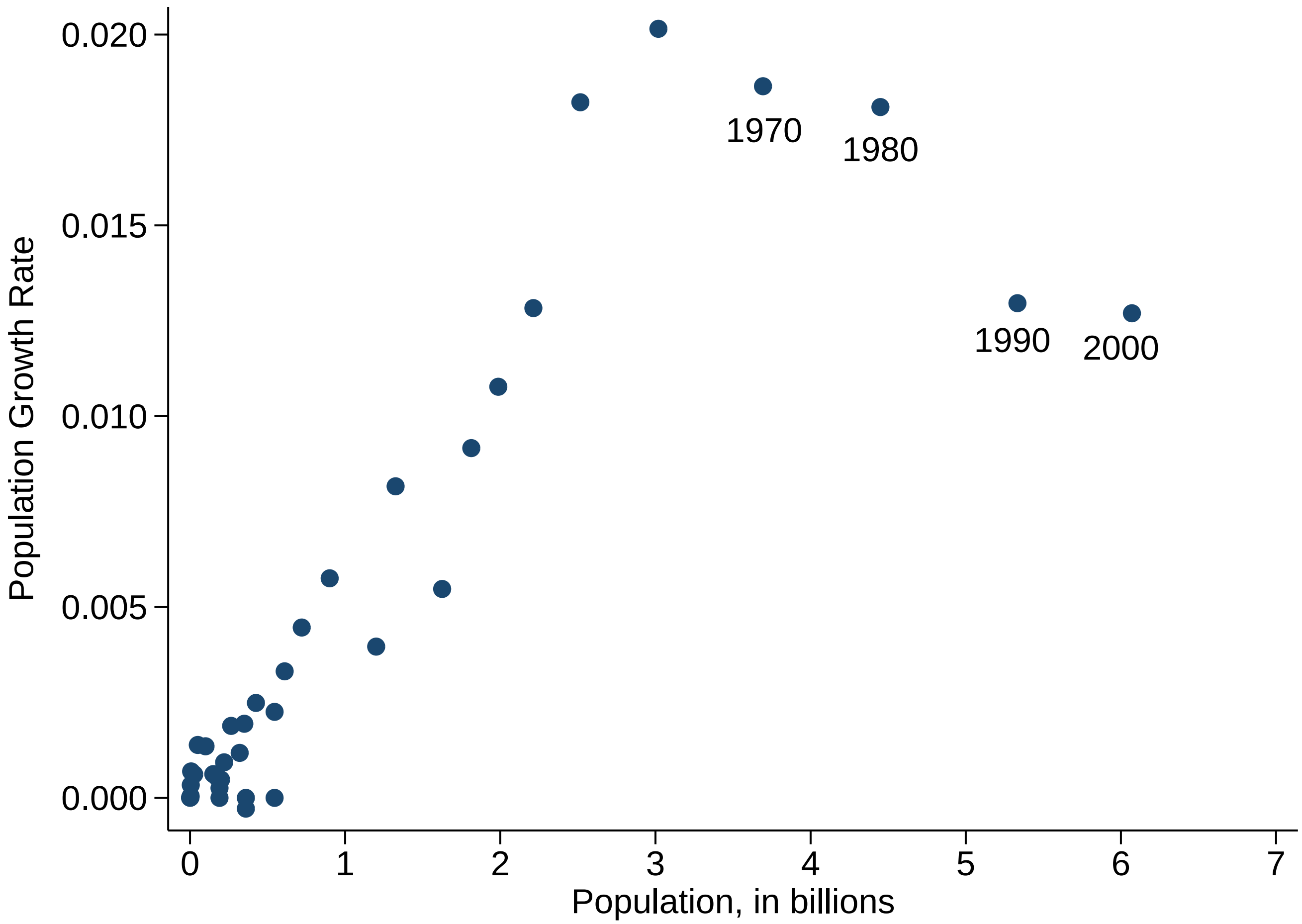
$$\frac{\dot{B}}{B} = \nu L \tag{11}$$

and combine with the steady state condition that $\dot{L}/L = g = 1/\beta(\dot{B}/B)$ gives us

$$\frac{\dot{L}}{L} = \frac{\nu}{\beta} L \tag{12}$$

Implication is that the growth rate of popuation (which depends on technological growth) is positively related to the *size* of the population (which drives technological growth).

Population Growth Over Very Long-Run



Transition to Modern Growth

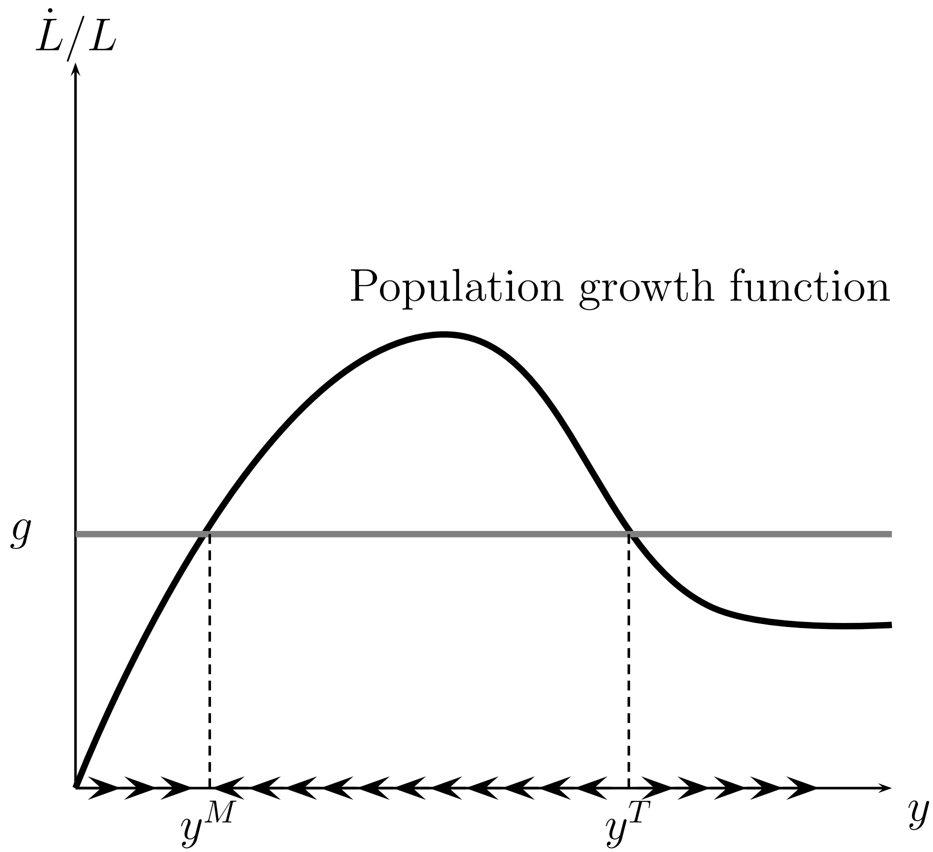
Kremer model works for most of history.

- Breaks down in 20th century
- Population growth has not spiraled up continuously
- Rate of technological change has not spiraled up continuously

We need to adapt the model to capture the endogenous response to population to higher output per worker (Demographic Transition) that ensures world ends up with constant technological change (as in the Romer/Schumpeter models).

More Realistic Population Growth Function

Rather than linear $\frac{\dot{L}}{L} = \theta(y - \underline{c})$, let population growth look like this:



From Malthusian Stagnation to Modern Growth

The transition is driven by the interplay of population size and technological change

- Malthusian era:
  - Population, L, is small, so technological growth, g, is low
  - Low g means low y^M, Malthusian steady state income
- Post-Malthusian era:
  - Population, L, reaches a sufficiently large size that g gets big (above max of population growth function)
  - This accelerates population growth $\dot{L}/L$ while also allowing y to rise continuously
- Modern Growth era:
  - Population, L, is still very large, but B has risen so much that g slows down
  - Population growth, $\dot{L}/L$, slows down as economy gets richer
  - Eventually population growth levels off at some constant, where we are in Romer/Schumpeter model

Growth in Modern Growth Regime

Slightly different answer for long-run growth than Romer/Schumpeter. We know that steady state technology growth is

$$\frac{\dot{B}}{B} = \frac{\lambda}{1-\phi} n^* \tag{13}$$

where n^* is the long-run population growth rate.

But growth in output per worker is

$$\begin{aligned} \frac{\dot{y}}{y} &= \frac{\dot{B}}{B} - \frac{1}{\beta}\frac{\dot{L}}{L} && (14) \\ &= \frac{\lambda}{1-\phi} n^* - \frac{1}{\beta} n^* && (15) \\ &= \left(\frac{\lambda}{1-\phi} - \frac{1}{\beta}\right) n^* && (16) \end{aligned}$$

There is this additional "drag" on economic growth of $1/\beta$ which captures the influence of the fixed factor of production, X. So long as $\lambda/(1-\phi)$ is large enough, innovation outweights Malthusian effects of fixed resource.

Structural Change

Reason that innovation tends to win is that as we get richer, β falls. That is, fixed resources (land, energy) become less important in production. We'll revisit this in another chapter.