

# Credit markets, Inequality and growth w/ debt overhangs

**ECGA 6470: Economic Growth Development**  
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## Imperfect Credit markets Aghion and Williamson, 1999

Following Aghion and Bolton (1997), we introduce moral-hazard considerations as the explicit source of credit-market

imperfections into the AK with overlapping-generations framework developed above. Specifically, we again assume the existence of a continuum of non-altruistic, overlapping-generations families, indexed by  $i \in [0, 1]$ . The utility of individual  $i$  in generation  $t$  is

$$U_t^i = d_t^i - c(e_t^i), \tag{6}$$

where  $d_t^i$  denotes individual  $i$ 's second-period consumption (for simplicity we assume that individuals consume only when old),  $e^i$  is the non-monetary effort incurred by individual  $i$  when young, and  $c(e^i) = A(e^i)^2/2$  denotes the non-monetary cost of effort. The parameter  $A$  still measures productivity on the current technology. As before, the human-capital endowment of individual  $i$  is taken to be an idiosyncratic proportion of average knowledge at date  $t$ , that is,  $w_t^i = \epsilon_t^i \cdot A_t$

## Aghion and Williamson, 1999

The production technology involves an extreme form of U-shaped average cost curve with respect to capital investments, namely:

- (a) the production activity requires a *fixed* and indivisible capital outlay equal to  $k_t^i = \varphi \cdot A_t$ ;
- (b) conditional upon the required investment  $\varphi \cdot A_t$  being made at date  $t$ , the output from investment in this technology is uncertain and given by

$$y_t^i = \begin{cases} \sigma \cdot A_t & \text{with probability } e_t^i \\ 0 & \text{with probability } 1 - e_t^i \end{cases}$$

where  $e_t^i$  is individual  $i$ 's effort at date  $t$ . We assume that second-period outcomes  $y_t^i$  are independently identically distributed across individuals of the same generation.

## Aghion and Williamson, 1999

The source of capital-market imperfection will be moral hazard with limited wealth constraints (or limited liability), in other words, the assumption that:

- (a) efforts  $e^i$  are not observable;
- (b) a borrower's repayment to his lenders cannot exceed his second period output  $y_r^i$

## Aghion and Williamson, 1999

Consider the effort decision of an individual who does not need to borrow, that is, for whom  $w^i \geq \varphi A$ . The problem he faces is

$$\max_e \{e \cdot \sigma A - c(e)\},$$

which gives the first-best level of effort,  $e^* = \sigma$ .

An agent with initial endowment  $w^i < \varphi A$  needs to borrow  $b^i = \varphi A - w^i$  in order to invest. Let  $\rho$  be the unit repayment rate owed by individual  $w^i$ . Hence, he chooses effort  $e^i$  to maximize the expected second-period revenue net of both repayment to the lenders and effort cost, namely

$$\begin{aligned} e^i &= \max_e \{e(\sigma A - \rho(\varphi \cdot A - w^i)) - c(e)\} \\ &= e(\rho, w^i), \end{aligned} \tag{7}$$

where  $e(\rho, w^i) = \sigma - \rho(\varphi - w^i/A)$  is less than the first-best effort  $e^*$ , and is decreasing in  $\rho$  and increasing in  $w^i$ .



The growth rate of the economy is given by

$$g = \ln \frac{\sigma A \cdot \int e^i di}{A}$$
$$= \ln \sigma + \ln \int_0^1 e^i di, \quad (8)$$

with efforts  $e^i \leq \sigma$ . If either (a) or (b) were violated, then the first-best effort would automatically be elicited from *all* individuals no matter what their human-capital endowments were. The growth rate would then be unaffected by redistribution and always be equal to  $g = \ln \sigma^2$ . This corresponds to nothing but the case of *perfect* capital markets, that is of capital markets that do *not* suffer from incentive problems. When there are incentive problems, the more unequal the distribution of wealth is, that is, the larger the number of individuals with wealth below the threshold level  $\varphi A$ , the lower the aggregate level of effort will be. Consequently, inequality has a negative effect on both the income level and the growth rate.

We now have all the elements we need to analyze the incentive effects of redistribution. Because individuals with initial wealth  $w^i \geq \varphi A$  supply the first-best effort  $e^* = \sigma$ , raising a lump-sum tax  $t^i < w^i - \varphi A$  on the endowment of each such individual and then distributing the total proceeds among borrowers:

- (i) will not affect the effort  $e^*$  supplied by the wealthy, whose *after-tax* endowments remain strictly above the required fixed cost  $\varphi A$ ;
- (ii) will increase the effort supplied by any subsidized borrower.

The above redistribution scheme will then have an unambiguously positive *incentive* effect on growth, as efforts  $e^i$  either increase or remain constant as a result of redistribution.

To see how inequality induces free-riding consider the following set up. Suppose that the economy gives birth to only two individuals each period, and that these two individuals (who both live for two periods) need to join forces (that is, to pool their initial resources) in order to produce. Let  $\bar{w}_t = \bar{w}A_t$  and  $w_t = wA_t$  denote the initial endowment of the richer and the poorer of these two individuals. As above, we denote by  $\varphi \cdot A_t$  the fixed cost of the project initiated at date  $t$ , and we assume that

$$\bar{w} + w \geq \varphi > \bar{w} > w.$$

In other words, the project requires the financial participation of both individuals in order to be implemented at all.



## Inequality and growth a [JEL survey article](#) Aghion et. al, 1997 [Chapter 1](#) or Penalosa's lecture notes

The basic incentive argument carries over to the aggregate economy when agents are identical and/or capital markets are perfect, as shown by Rebelo (1991). In a Ramsey-Cass-Koopmans growth model with perfect capital markets, the rate of growth of individual consumption is given by

$$g = \frac{r - \rho}{\sigma},$$

where  $\rho$  is the intertemporal discount rate,  $r$  the *after-tax* interest rate and  $\sigma$  the intertemporal elasticity of substitution. If

**Same Solow-Swan model but now we add up all individual agent to get total stock of knowledge, page 1622 of [Aghion et al. 1999](#)**

More formally, suppose that when individual  $i$  invests an amount of physical or human capital  $k_{i,t}$  at date  $t$ , production takes place according to the technology

$$y_{i,t} = A_t k_{i,t}^\alpha,$$

where  $0 < \alpha < 1$ .  $A_t$  is the level of human capital or technical knowledge available in period  $t$ , and it is common to all individuals. The level of technology is endogenous, as the economy exhibits both learning-by-doing and knowledge spillovers. Learning-by-doing means that the more an agent produces one period, the more she learns, and hence the greater the level of knowledge available in the next period. The presence of spillovers implies that the learning done by one individual affects the level of technology



**Inequality:  
entire stock  
of wealth is  
distributed  
randomly in  
each period**

$$w_{i,1} = a \cdot \varepsilon_{i,t}$$

**(see ACG  
page 1623**

To see how investments are determined, consider an economy with only one good that serves both as capital and consumption good. There is a continuum of overlapping-generation families, indexed by  $i \in [0,1]$ . Each individual lives for two periods. The utility of an individual  $i$  born at date  $t$  is given by  $U_t^i = \log c_{i,t} + \rho \cdot \log c_{i,t+1}$ , where  $c_{i,t}$  and  $c_{i,t+1}$  denote current and future consumption respectively. Individuals differ in their initial endowments. In order to abstract from intergenerational transfers and bequest decisions, suppose that initial endowments are randomly determined at birth. Let the endowment of individual  $i$  upon birth at date  $t$  be given by

$$w_{i,t} = a \cdot \varepsilon_{i,t},$$

where  $a$  is a constant and  $\varepsilon_{i,t}$  is an identically and independently distributed random variable, with mean  $\frac{1}{a}$ .

**Learning by doing, and spillovers, so for each individual  $A_t$  is now the sum of all knowledge, that is,  $y_{t-1}$  (see [Aghion et al p 1623](#))**

$$A_t = \int y_{i,t-1} di = y_{t-1}.$$

That is, the accumulation of knowledge results from past aggregate production activities.

As a result of learning-by-doing, growth depends on individual investments. The rate of growth between period  $t-1$  and period  $t$  is given by  $g_t = \ln(y_t/y_{t-1})$ , that is,

$$g_t = \ln \frac{\int A_t k_{i,t}^\alpha di}{A_t} = \ln \int k_{i,t}^\alpha di,$$

It can then be expressed simply as

$$g_t = \ln E_t[k_{i,t}^\alpha],$$

where  $E_t[k_{i,t}^\alpha]$  is the mathematical expectation over the output generated by individual investment levels at date  $t$ .



**With credit  
market  
imperfection  
asset  
redistribution  
problem  
returns (see  
page 1625)**

fections, based on Aghion and Bolton (1997). Specifically, we assume again the existence of a continuum of nonaltruistic, overlapping-generation families, indexed by  $i \in [0,1]$ . The utility of individual  $i$  in generation  $t$  is

$$U_t^i = c_{i,t} - h(e_{i,t}),$$

where  $c_{i,t}$  denotes individual  $i$ 's second-period consumption (for simplicity we assume that individuals consume only when old),  $e_{i,t}$  is the nonmonetary effort incurred by individual  $i$  when young and  $h(e_{i,t}) = A_t e_{i,t}^2 / 2$  denotes the nonmonetary cost of effort. The parameter  $A_t$  still measures productivity on the current technology. The endowment of individual  $i$  is taken to be an idiosyncratic proportion of average knowledge at date  $t$ , that is,  $w_{i,t} = \varepsilon_{i,t} \cdot A_t$ .