One controversial issue is the effect of government spending on economic growth. Does infrastructure spending enhances productivity and increase economic growth or "crowd out" private spending reducing growth and efficiency? The empirical evidence is mixed: some studies find public and private investment is complementary while others find public consumption spending and even public investment reduces private investment. Some insight into why the empirical evidence might be mixed can be derived from a simple model of economic growth proposed by Barro (1990).

Starting with the Robelo or "AK" endogenous growth model, Barro suggests human plus physical capital ("K") can be made more productive by adding just the the right amount of public infrastructure services ("G"). Using the Cobb-Douglas production function $Y = G^{\alpha}K^{ß}$, with constant returns $\alpha + ß = 1$ so that $ß = 1 - \alpha$. The catch is that spending on G must be finance by an income tax on income generated by physical + human capital owned by private agents, K. So assuming an government budget that must be balanced in the long run, $G = \tau Y$ where τ is the income tax rate. Using a constant relative risk aversion utility function (see the "Three growth Models" handout), the per capita growth rate is then,

$$\boldsymbol{\gamma = (1/\theta)(A - \rho),}$$

where $1/\theta$ is consumers' constant IES and ρ is the rate of time preference or discount. In the "AK" model A is just the marginal product of capital, f_k. Now adding government changes the model in two ways. First private "after tax" income from or marginal productivity of capital is now $(1 - \tau)f_k$. Second, having the right amount in infrastructure g makes capital more productive. Hence our modified growth equation becomes,

$$\boldsymbol{\gamma = (1/\theta[(1 - \tau)f_k - \rho]}$$

Clearly an increase in the tax rate now has two opposite effects on growth: it reduces the income available to the private sector but over a certain range more public investment raises the MPK, f_k. Substituting τy into the production function (lower case y is per capita GDP, γ is per capita infrastructure stock, etc.) and a little algebra yields the new $f_k = A^*$ and growth is now,

$$\boldsymbol{\gamma = (1/\theta)(A^* - \rho) \text{ where } A^* = (1-\alpha)\, A^{1/(1-\alpha)} * \tau^{\alpha/(1-\alpha)}.}$$

Clearly, A^* rises with τ even as $(1 - \tau)$ or after tax income falls. The net result is that for while raising taxes and spending in γ raises the growth rate, but after a certain point raising τ further reduces growth. As it happens α is the optimal tax rate (government spending share of national income is. Any increase of the tax rate above α, a technology parameter, reduces growth. This may explain why some authors find growth is affected negatively by government spending while others find a positive effect (see for example Barro, 1991 who finds a positive effect). A similar approach is used by Basu and McLeod (1991) to incorporate terms of trade effects into an endogenous growth model.

Further Reading:

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Barro's model of government spending and taxation

Barro (1990) suggests a simple endogenous growth model with government. Departing from the standard characterization of government consumption financed by distortionary taxes as in Easterly (2005) in Barro's model public investment (roads, ports, sanitation, schools, etc.) complements private investment. Government spending is finance by a straight income tax. Since public investment raises the productivity of private investment, higher taxes can be associated with an increase or a decrease in overall growth. But because the private sector ignores the additional tax revenues and public investment generated by its private investment, it tends to invest too little. Because it is a close cousin of the AK model this model provides a very tractable framework for many problems such as imported investment goods in Basu and McLeod (1992). Barro's starts with the AK model and adds public investment or capital g:

$y = Ak^{1-\alpha}g^{\alpha}$ where $g = \tau y$, where g is government spending per capita is determined by the tax rate τ on income per capita y since the budget is always balanced. Using the government budget constraint to substitute for g in (1A) we obtain, $y = A(\tau y)^{\alpha}k^{1-\alpha}$ bringing y^{α} over to the LHS, $y^{1-\alpha} = A\tau y^{\alpha}k^{1-\alpha}$ raising both sides to 1-α yields $y = \left(A^{\frac{1}{1-\alpha}}\tau^{\frac{\alpha}{1-\alpha}}\right)k = A^*(\tau)k$ where the new MPK in brackets $A^*(\tau)$ plays a role analogous the A in the AK model, except that it depends on the tax rate τ. One might think the growth rate $\gamma = 1/\theta\ [A^* - \rho]$ where again intertemporal elasticity of substitution is $1/\theta$ and ρ is the discount rate. But recall that the private sector only receives after tax income on capital investment, or $(1-\tau)(1-\alpha)A^*(\tau)$. Note that government has two effects on private income, reducing after tax income $(1-\tau)$ but also raising the marginal product of capital $A^*(\tau)$ due the contribution of g to total output for any given private capital stock k. At some point these two effects balance out and we obtain the growth maximizing tax rate which is α, the optimal share of g implied by our Cobb-Douglas production function (analogous to the golden rule savings rate $s = \alpha$). The growth maximization problem has another twist however. The private sector does not take into account that the taxes it pays on investment income increase g (or perhaps they don't believe it) it will under save/invest and the growth rate will be too low. The socially optimal growth rate is $(1-\alpha)A^*(\tau) > (1-\tau)(1-\alpha)A^*(\tau)$ so the decentralized growth rate is lower that the optimal or "central planners" growth rate. Note also that if $\tau = 0$, $g = 0$ and we have a public investment poverty trap as the private MPK = 0 and the growth rate is negative. Of course if the government takes the whole national product so that $\tau = 1$ the growth rate also goes to zero since there is no private investment. The optimal tax rate is obviously somewhere in between 0 and 1.

To solve the households (and planners) optimal growth problem more explicitly, assume infinitely lived households own firms and produce output using a production function that benefits from per capita infrastructure investment g (roads, ports, dams, etc). Households maximize a constant relative risk aversion or CRRA utility $u(c_t)$,

$$\max \int_0^\infty \frac{c_t^{1-\theta}}{1-\theta} e^{-\rho t} dt \quad \textit{subject to} \quad \dot{k}_t = (1-\tau) y_t - c_t \text{ where } y_t = A k_t^{1-\alpha} g_t^{\alpha} \text{ and } 0 < \alpha < 1.$$

Per capita public spending on infrastructure g is financed by a proportional tax on income so $g = \tau y$. The government does not borrow so the budget is always balanced. All variables are per capita but note that labor is not required to produce y but we assume k includes human as well as physical capital. To solve maximization problem of a representative, infinitely lived private agent who controls k and consumes c taking g and τ as given (set by the government) we can use a present value Hamiltonian solving for the present value of consumption subject to the accumulation constraint in brackets:

$$H = \frac{c_t^{1-\theta}}{1-\theta} e^{-\rho t} + \mu\left[(1-\tau) y_t - c_t\right] \textit{ where } y_t = A k_t^{1-\alpha} g^{\alpha}.$$

Since the private sector only controls k, we differentiate H with respect k_t and c_t only to obtain the first order conditions (FOCs):

$$H_c: \quad c_t^{-\theta} e^{-\rho t} - \mu = 0 \qquad (1)$$
$$H_k: \quad \mu\left[(1-\tau)(1-\alpha) A k^{-\alpha} g^{\alpha}\right] = \dot{\mu} \qquad (2)$$
$$H_\mu: \quad \left[(1-\tau) y_t - c_t\right] = \dot{k} \quad \textit{where again, } y_t = A k_t^{1-\alpha} g^{\alpha}$$

The transversality condition is $\lim_{t\to\infty}\left[\mu(t)\cdot k(t)\right] = 0$ which the dynamic version of the Inada conditions. To solve for the growth rate of consumption first take log of (1) to obtain:

$-\theta * \ln c_t - \rho * t = \ln \mu$. Taking the time derivative of this equation yields:

$$-\theta \cdot \frac{\dot{c}}{c} - \rho = \frac{\dot{\mu}}{\mu} \qquad (3)$$

Dividing both side of (2) by μ gives $\left[(1-\tau)(1-\alpha) A k^{-\alpha} g^{\alpha}\right] = \frac{\dot{\mu}}{\mu}$. Using this expression to replace the RHS of (3) yields: $\left[(1-\tau)(1-\alpha) A k^{-\alpha} g^{\alpha}\right] = -\theta \cdot \frac{\dot{c}}{c} - \rho$

Solving for $\frac{\dot{c}}{c}$ we get the decentralized competitive solution:

$$\frac{\dot{c}}{c} = \frac{1}{\theta}\left(\underbrace{\left[(1-\tau)(1-\alpha) A k^{-\alpha} g^{\alpha}\right]}_{MPK} - \rho\right) \quad or \quad g = \frac{\dot{c}}{c} = \frac{1}{\theta}\left(A^* - \rho\right) \textit{ where } A^* = (1-\tau)(1-\alpha) A k^{-\alpha} g^{\alpha} \qquad \textbf{(4)}$$

This expression says the growth in consumption depends on (i) the gap between the marginal product of capital and the rate of time preference, ρ and (ii) on the IES $1/\theta$. The higher the θ, the more value individuals place on smoothing consumption. The higher θ is, the higher return to capital (MPK) it takes to encourage investment which is what raises the growth rate of consumption (and output y).

It appears that the blue MPK term in (4) depends on k, but it does and should not since this an AK class endogenous growth. To see this recall that $g = \tau Y$ so that $g = \tau \cdot Ak^{1-\alpha} g^{\alpha}$, solving for g yields,

$g = \left(\tau \cdot Ak^{1-\alpha}\right)^{\frac{1}{1-\alpha}} = (\tau \cdot A)^{\frac{1}{1-\alpha}} k$ which implies that $g^{\alpha} = (\tau \cdot A)^{\frac{\alpha}{1-\alpha}} k^{\alpha}$. Substituting g^{α} into the blue MPK term above causes the k terms to drop out leaving, **MPK =** $(1-\tau)(1-\alpha)A^{\frac{1}{1-\alpha}}\tau^{\frac{\alpha}{1-\alpha}}$ **= A*(τ)**

Hence the after-tax marginal product of capital calculated above, **A*(τ)**, plays the same role as the A in the AK model (except that it depends on the tax rate). The expression consumption growth equation (4) shows that government affects the marginal product of capital through two channels: increases in g raise the MPK up to a point, but taxes always reduce the private return to capital, the task of a good government is balance these two effects:

$$MPK = \underbrace{(1-\tau)}_{negative\ effect\ of\ taxation} (1-\alpha)Ak^{-\alpha} \underbrace{g^{\alpha}}_{positive\ effect\ of\ public\ services}. \tag{5}$$

To solve the model from the point of view of a benevolent growth maximizing government (central planner) we know that the planner satisfies the condition δy/δg =1 ("marginal product of government spending" or MPG). The government should provide public investment or services until the MPG equals one. So the MPG must equal government spending *g:*

$$\underbrace{\alpha Ak^{1-\alpha} g^{\alpha-1}}_{MPG} = \underbrace{g}_{spending}$$

Solving for g we find that $1/\alpha$ = (y/g)= 1/τ, so that **α = τ** when MPG =1 .

The key difference between the decentralized and the centralized solution, is that the firm responds to the the after-tax private marginal product MPK= (1-τ)∂y/∂k while the planner takes into account the social marginal product (∂y/∂k): additional private income also raises taxes and g as well as k. To get the social planner's growth rate, eliminate the (1-τ) from (5) to find the planner's MPK and we get that MPK = $(1-\alpha)A^{\frac{1}{1-\alpha}}\tau^{\frac{\alpha}{1-\alpha}}$ where substituting **α = τ** yields MPK = $(1-\alpha)A^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}$

So the planner should set the tax rate to maximize growth rate and the red MPK,

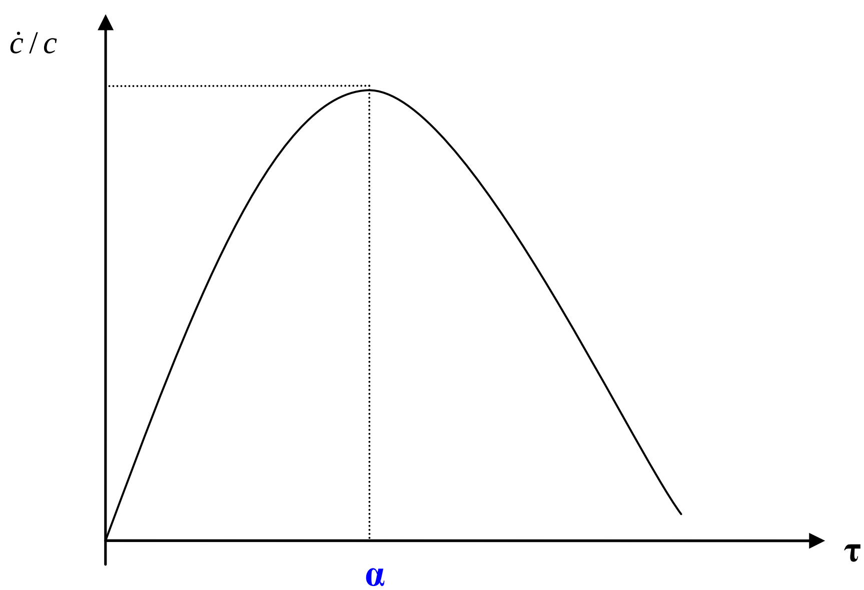
$$\frac{\dot{c}}{c} = \frac{1}{\theta}\left(\underbrace{\left[((1-\alpha)A^{\frac{1}{1-\alpha}}\tau^{\frac{\alpha}{1-\alpha}}\right]}_{MPK} - \rho\right). \tag{6}$$

Note the MPK in the decentralized solution is (1-τ)(∂y/∂k), it is smaller than the social marginal product ∂y/∂k, because of the tax rate. This gap between social and private returns leads to a lower growth rate in the decentralized solution summarized by (4) compared to the socially optimal growth shown in eq. (6). Note however, that whether the relevant growth rate is dictated by (4) or (6), the growth maximizing tax rate is **τ** = α**.** For the decentralized solution, a benevolent government would want to maximize the utility of individuals with a certain size of the government, *g*. To find this, we differentiate the following growth rate (combination of 4 and 5) with respect to **τ:** $\frac{\dot{c}}{c} = \frac{1}{\theta}\left(\underbrace{\left[(1-\tau)(1-\alpha)A^{\frac{1}{1-\alpha}}\tau^{\frac{\alpha}{1-\alpha}} \right]}_{MPK} - \rho \right)$

Differentiating: $\partial(\dot{c}/c)/\partial\tau = \left[-1\left((1-\alpha)A^{\frac{1}{1-\alpha}}\tau^{\frac{\alpha}{1-\alpha}} \right) \right] + (1-\tau)(1-\alpha)\left(\frac{\alpha}{1-\alpha} \right)A^{\frac{1}{1-\alpha}}\tau^{\frac{-1}{1-\alpha}} = 0$

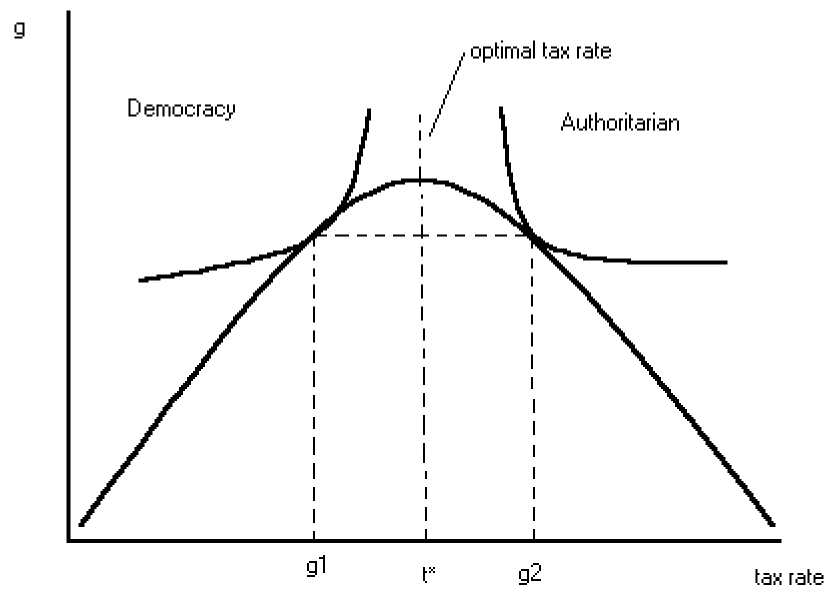
Simplifying: $\left[\left((1-\alpha)\cancel{A^{\frac{1}{1-\alpha}}}\tau^{\frac{\alpha}{1-\alpha}} \right) \right] = (1-\tau)\left(\frac{\alpha}{1-\alpha} \right)(1-\alpha)\cancel{A^{\frac{1}{1-\alpha}}}\tau^{\frac{-1}{1-\alpha}}$,

$(1-\alpha)\tau = \alpha(1-\tau)$, which can only hold if $\tau = \alpha$. The condition that **τ =α** corresponds to the golden rule savings rate in the Solow model (recall that s = α in problem set #1) here the tax rate is the savings rate for public owned capital, so α is the optimal tax rate in both solutions. Graphically, the optimal tax rate **τ** = α is



A note on Democracy and Economic Growth

There is large (inconclusive) literature on whether democracies have higher growth rates. Assuming taxpayers vote and paying taxes is painful (distortionary) the optimal tax rate for a democracy is likely to be a bit lower than with no frictions (lower than α in the above example). If voters (taxpayers) have no influence on the other hand, and authoritarian rulers benefit directly from higher tax revenues (e.g. higher rents or salaries) the optimal tax rate for an authoritarian regime is likely to be higher than α. The implication of these arguments is summarized in the diagram below, the democratic and authoritarian regime and may have similar growth rates, but the government sector is likely to be smaller with a democratic regime. This would be an interesting hypothesis to test, all else equal.



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